Managing Weather Risk with a Neural Network-Based Index Insurance

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Abstract

Weather risk affects economy, agricultural production in particular. Index insurance is a promising tool to hedge against weather risk, but current piecewise-linear index insurance contracts face large basis risk and low demand. We propose embedding a neural network-based optimization scheme into an expected utility maximization problem to design the index insurance contract. Neural networks capture highly nonlinear relationship between the high-dimensional weather variables and production losses. We endogenously solve for the optimal insurance premium and demand. This approach reduces basis risk, lowers insurance premium, and increases farmers' utility. Our approach can be generalized to design other financial products.

Keywords: neural networks; weather risk; index insurance; basis risk; utility maximization

JEL code: C45, G11, G22, G52, O13, Q54

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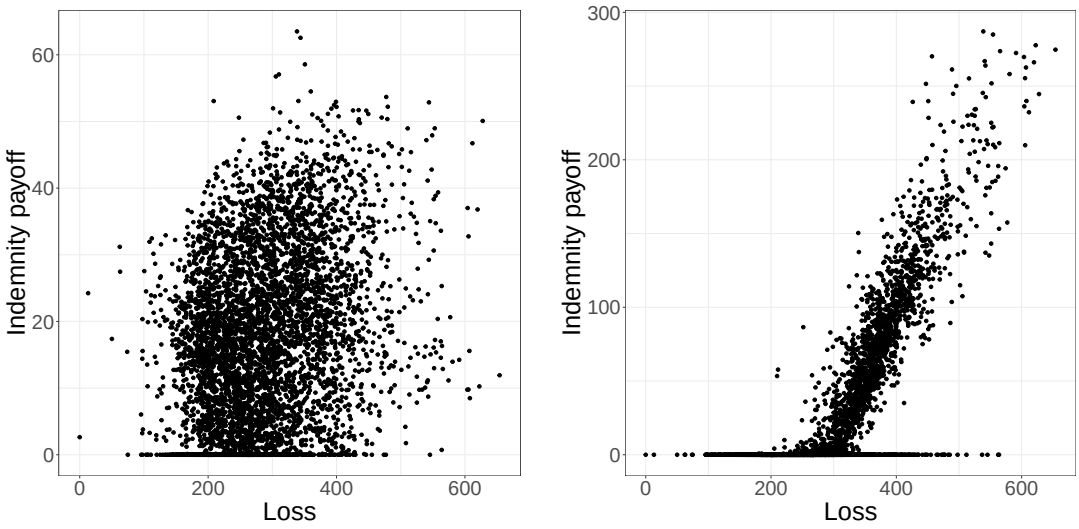
1 Introduction

Climate change and weather risk affect economy and livelihood over large scales (Nordhaus, 2019; Hong et al., 2020), especially for agricultural production which depends on weather conditions (Fisher et al., 2012; Lesk et al., 2016). For example, USDA (2014) estimates that 70%-90% of agricultural production loss can be attributed to adverse weather. In practice, weather index insurance can be employed to hedge against weather risk (Turvey, 2001). The payoff of weather index insurance is exclusively based on some prespecified weather indices instead of the actual losses incurred to the insureds. Hence it avoids the high administration costs, adverse selection and moral hazard issues associated with conventional indemnity-based insurance. While promising, current index insurance faces low demand (Cole et al., 2013; Clarke, 2016).

Large basis risk, the risk that the underlying indices and actual losses are mismatched, contributes to the low insurance demand (Cummins et al., 2004; Clarke, 2016; Jensen et al., 2016).[1] Most current index insurances are based on low dimensional weather indices and use linear-type payoff functions (Mahul and Skees, 2007; Giné et al., 2007). Panel (a) of Figure 1 plots the payoffs of a currently used linear index insurance based on a temperature index, exhibiting large basis risk. Moreover, given the large basis risk, the remaining uninsured risk could be high, therefore the current insurance appears to be not cost effective for farmers (Jensen et al., 2016), which further hinders farmers' participation. This paper attempts to address the above two issues to design better index insurance contracts.

We aim to reduce the basis risk with two considerations. First, crop production depends on high-dimensional weather conditions. We should carefully select and include a sufficiently large number of weather variables when constructing the contract. Second, because of biological reasons, crop production depends on weather conditions in a highly nonlinear way

[1] Other factors also affect the demand of index insurance, including asymmetric information (Hartman-Glaser and Hébert, 2020), ambiguity aversion (Bryan, 2019), farmers' past insurance payout experiences (Cole et al., 2014; Cai et al., 2020), and insurance literacy (Cai et al., 2020; Gaurav et al., 2011).



(a) A piecewise linear contract (b) An optimal NN-based contract

Figure 1: **Payoffs of index insurance and actual losses.** These panels plot the insurance payoffs against actual losses to illustrate the basis risk. Panel (a) plots a piecewise-linear insurance contract based on a temperature index, which is commonly used in the current practice. Panel (b) plots an NN-based index insurance contract.

(Schlenker and Roberts, 2009; Lobell et al., 2011; Rigden et al., 2020). Tapping into recent advances in machine learning, we propose a neural network-based (NN-based) index insurance design.[2] Neural networks help to capture high-dimensional, nonlinear, and complex interactions between weather indices and production losses. Specifically, we embed an NN-based scheme into an expected utility maximization problem with budget constraints to capture farmers' insurance demand.

Moreover, in addition to crop yield prediction, we endogenously determine the equilibrium insurance premium and demand, while the current literature typically assumes exogenously given insurance premium (using burning cost or via an ad hoc risk loading). This economic

[2]Other machine learning techniques might be useful, e.g., reinforcement learning. We opt to the neural network approach for two reasons. First, NN provides satisfying performance and computing speeds (Gu et al., 2020; Feng et al., 2021). Second, it is impossible to forecast weather conditions and production losses one-year ahead, given current atmospheric models (Voosen, 2019). Therefore, farmers cannot time the index insurance market at the annual frequency. Nevertheless, reinforcement learning could be helpful if we consider farmers with bounded rationality, e.g., farmers are short-sighted and only learn from the recent experiences. We avoid designing the insurance contract based on behavioral bias as this might introduce some litigation risks in practice.

mechanism incorporates strategic interaction between farmers and insurers, which is important in two ways. First, it allows for a fair evaluation of welfare improvement for farmers. Second, once we consider the responses of both insureds and insurers in an equilibrium setting, it improves the cost effectiveness and feasibility of the designed insurance contract.

We apply this NN-based index insurance to corn production in Illinois. We first explore various NN structures to identify the optimal neural network, which has three hidden layers (64, 64, and 16 neurons in each layer). Results show that the NN-based index insurance improves farmers' utility because this contract reduces their exposure to wealth variations, especially the extreme downside risk, and it is cost effective. For example, using a constant absolute risk aversion (CARA) utility, we see that the optimal NN-based index insurance improves farmers' utility by 14.35% for the test sample. Certainty equivalent wealth (CEW) improves by \$22,767.36 for the test sample for an average farm, which is about 4.98% of her wealth. The optimal NN-based index insurance has a premium of \$28.72/acre, which is 63% of the current corn insurance premium (\$45.50/acre) in Illinois, making the contract very attractive. Panel (b) of Figure 1 plots the payoffs of the optimal NN-based index insurance policy. Comparing Panels (a) and (b), we see that the optimal NN-based contract effectively reduces basis risk. In fact, the payoff function of our proposed NN-based index insurance is very close to a conventional indemnity-based insurance with a deductible.

We dig deeply in two ways to understand the economic mechanism of the superior performances of the NN-based index insurance contract. First, following Cong et al. (2021a) and Ribeiro et al. (2016), we perform sensitivity analyses to improve model transparency and interpretability. Interestingly, we find that some weather indices outside growing seasons (the growing season in Illinois is normally from May to October for corns) are important, e.g., dew point temperature and maximum temperature in December, whereas these variables are overlooked by the linear insurance contract in current practice. This demonstrates the power of neural networks to extract important information from a large set of input factors.

Second, we compare the performances of NN-based index insurance with other insur-

ance contracts that are based on polynomial payoff functions, such as linear polynomials (the common practice nowadays), quadratic polynomials, and cubic polynomials. We also consider contracts using different numbers of weather indices as input factors. We find that including more weather variables and nonlinear terms improve farmers' utility. These results suggest the impressive performances of NN-based contract are indeed rooted in the contract's capability to extract complicated information from a large set of inputs, which are often nonlinear and nonmonotonic.

We further verify the robustness of NN-based index insurance in several ways. First, we perform an "out-of-state" tests, using major corn producing states adjacent to Illinois that have similar latitudes, namely, Indiana, Iowa, and Missouri. That is, we use Illinois data as the training sample and data from the three adjacent states as the test sample. Second, we consider the impacts of over-insuring constraint. Third, we examine the impacts of weather predictability on insurance design. Last, we consider the impacts of insurers' choices (such as various insurers' supply curves and exogenously given risk loading) and farmers' characteristics (such as different coverage levels, different risk aversion levels, time-varying risk aversion, and alternative utility functions).

Although effective, the NN-based contract is more complicated than linear contracts. Complexity aversion might hinder farmers from participating the NN-based insurance markets. We address this complexity concern in three ways. First, improving the interpretability of NN-based contracts and farmers' insurance literacy can boost farmers' trust in this product (Gaurav et al., 2011; Cai et al., 2020). Second, similar to Cai et al. (2020), we show that government subsidies can significantly increase farmers' participation. Last, to introduce this insurance product, we propose a hybrid contract which provides payoffs as the maximum of a linear contract and an NN-based contract as a feasible solution. Although suboptimal, such a hybrid contract benefits from easy interpretability of a linear contract and large basis risk reduction of the NN-based contract.

Our framework is flexible enough to be extended to other optimization problems. For

example, we extend our framework to the case of revenue insurance which protects both the production and corn price risks. We also apply our approach to the case where risk-neutral agent optimizes her tail risk, e.g., the value-at-risk (VaR), instead of expected utility maximization. This can be useful for corporate farms, internal risk management of insurance companies or other financial institutions who provide credit to farmers.

This paper contributes to the growing literature applying machine learning in finance (see Lehalle and Capponi (2022) for a recent synthesis). Various machine learning algorithms have been examined to construct optimal portfolio, identify return factors, and evaluate model performances (Alsabah et al., 2020; Bianchi et al., 2021; Bryzgalova et al., 2021; Cong et al., 2021a,b; Feng et al., 2020, 2021; Gu et al., 2021, 2020). Machine learning is also applied to risk management, including default risk, mortgage risk, credit risk, and loan risk (Hurlin et al., 2018; Sirignano and Giesecke, 2019; Sirignano et al., 2021). Our paper proposes a general framework of applying neural networks to design financial products. In particular, we use it to design index insurance for better managing weather risk.

This paper also contributes to the index insurance literature. Several studies conduct field studies to investigate the demand and efficiency of index insurance (see, e.g., Cole et al., 2013, 2014; Chantarat et al., 2017; Casaburi and Willis, 2018). Clarke (2016) and Jensen et al. (2016) show that basis risk causes the low demand of index insurance. Assa and Wang (2020) design index insurance on agricultural goods which provides a high Sharpe ratio. Our paper illustrates the promise of an NN-based approach to reduce the basis risk and improve the demand of index insurance.

Our paper is closely related to the literature that uses machine learning techniques to improve crop yield predictions. For example, You et al. (2017), Crane-Droesch (2018), and Newlands et al. (2019) use deep learning models to predict agricultural yields. Complementing the literature, our paper applies the NN model to predict crop yields and importantly, it considers the endogenous pricing of the insurance contract, which affects insurance demand directly and basis risk indirectly. This is important economic mechanism overlooked in the

previous research.

This paper broadly belongs to the climate finance literature (see Hong et al. (2020) for a comprehensive overview). In particular, some papers explore financial solutions to manage weather risk. For example, Engle et al. (2019) consider climate risk hedging with mimicking portfolios. Our paper adds to the literature by proposing a new design of the weather-related insurance contract to manage weather risk.

The rest of this paper proceeds as follows. Section 2 introduces the model and assumptions. Section 3 discusses the NN-based approach for solving the optimal index insurance contract. Section 4 presents the empirical performance of the NN-based insurance policy. Section 5 provides robustness checks. Section 6 discusses several extensions of the model. Section 7 concludes.

2 The model

Consider a typical farmer who would like to hedge her exposure to weather risk, subject to certain budget constraints. Suppose the farmer has an initial wealth of w_0 and faces a random production loss of Y during a year. We assume there is an index insurance contract with a payoff determined by a p-dimensional random vector of indices, $\boldsymbol{X} = (X_1, X_2, ..., X_p)$. More specifically, the index insurance payoff is $I(\boldsymbol{X})$, where $I : \mathbb{R}^p \mapsto \mathbb{R}^+$ is the nonnegative payoff function. Denote the premium of the index insurance contract by $\pi(I)$, which is a functional of the indemnity function I. Then the terminal wealth of the farmer at the presence of the index insurance is $w_0 - Y + I(\boldsymbol{X}) - \pi(I)$. Our objective for the index insurance design is to select the optimal indemnity function I so that the policyholder's expected utility, $\mathrm{E}\,(U)$, is maximized under certain budget constraints. In this paper, we focus on designing index insurance against production losses, without taking into account crop price risk, that is, a yield insurance.[3] Specifically, the index insurance design problem can be formulated as the

[3]We extend this framework to revenue protection in Appendix M, which considers crop price risk.

following expected utility maximization problem:

$$\begin{cases} \sup\limits_{I\in\mathcal{I}} & \mathrm{E}\big(U[w_0 - Y + I(\boldsymbol{X}) - \pi(I)]\big) \\ \text{s.t.} & P_L \leq \pi(I) \leq P_U, \end{cases} \tag{1}$$

where $\mathcal{I} := \{I : \mathbb{R}^p \mapsto \mathbb{R}^+ | I \text{ is measurable}\}$ defines the feasible set of indemnity function I. The budget constraint of the farmer is given by P_L and P_U, that is, the lower and upper bounds of the premium level, which are given exogenously. The premium bounds indicate the price range that the farmer is willing to accept and they might be chosen as zero and her highest affordable price, respectively. The premium bounds also prevent abusive and speculative use of insurances in practice, especially when there are government subsidies.

In this paper, we employ the most commonly used premium principle both in the literature and in practice to determine the premium of the index insurance contract, the expectation premium principle.[4] That is, the index insurance premium $\pi(I)$ is proportional to the risk premium as follows:

$$\pi(I) = \lambda \mathrm{E}\left[I(\boldsymbol{X})\right], \tag{2}$$

where λ is the risk loading parameter and $\lambda \geq 1$. When $\lambda = 1$, $\pi(I)$ is called the actuarially fair premium. λ affects the insurance premium and insurer's profits, and also impacts farmers' index insurance demand. In this paper, we endogenously determine the equilibrium λ from the supply and demand curves of insurance contracts, which will be discussed in more details in Section 4.2.1. Such endogenous insurance premium takes into account the strategic interaction between farmers and insurers while the model solves for the utility maximization problem of farmers.

We need to compute the expected utility to solve problem (1). Instead of modelling the joint distribution of the loss Y and indices $\boldsymbol{X}$, we replace the expected utility with its empirical counterparts and directly search for the optimal index insurance. Specifically,

[4]In the actuarial science literature, a premium principle is a functional quantifying an insurable risk. See Young (2014) for a comprehensive introduction of premium principles.

for a random sample of $(\boldsymbol{X}, Y)$: $\{(\boldsymbol{x}_j, y_j)\}_{j=1,...,n}$, where $\boldsymbol{x}_j = (x_{j1}, x_{j2}, ..., x_{jp})$, after replacing quantities with sample statistics, problem (1) can be reformulated as the following minimization problem:

$$\begin{cases} \min\limits_{I \in \mathcal{I}} & -\frac{1}{n} \sum\limits_{j=1}^{n} U\big(w_0 - y_j + I(\boldsymbol{x}_j) - \pi_e(I)]\big) \\ \text{s.t.} & P_L \leq \pi_e(I) \leq P_U, \end{cases} \tag{3}$$

where $\pi_e(I)$ is the empirical counterpart of $\pi(I)$ and is given by $\pi_e(I) := \frac{\lambda}{n} \sum\limits_{j=1}^{n} I(\boldsymbol{x}_j)$.

3 A neural network-based solution

It is challenging to search for the optimal functional form of the insurance payout I in the feasible set $\mathcal{I}$ for the optimization problem (3). As a result, the existing literature has considered a certain restrictive feasible set $\tilde{\mathcal{I}}_0 \subset \mathcal{I}$. Step functions (where index insurance payment is triggered by some pre-defined events) and piecewise linear functions (an excess-of-loss-type contract where the triggered payment is a linear function of the index) are commonly used functional forms in the literature and in practice (see, e.g., Mahul and Skees, 2007; Giné et al., 2007). However, a restrictive feasible set generates contracts with large basis risk, which leads to market failure of index insurance (Clarke, 2016). Hence, it is necessary to allow for a larger set of feasible functional forms. However, the set of feasible functional forms cannot be too large. Otherwise, we will run into "overfitting" issues (see Appendix A for an example). Therefore, we need to choose a feasible set that balances flexibility and stability.

In this paper, we consider a feasible set which allows for nonlinear and nonmonotonic relationships. Specifically, we apply neural networks to search for the optimal contract in the expanded feasible set $\mathcal{I}_0 \subset \mathcal{I}$ (see Appendix B for comparisons of different feasible sets). NN is designed to capture high dimensions, nonlinearity, and complex interactions among input factors, which is particularly attractive for our research question. NN is sensitive to

critical details and insensitive to idiosyncratic outliers in the data, and can potentially be useful in achieving good flexibility-stability balance in solving for the optimal contract.[5]

3.1 Neural networks and feasible set

We consider a standard multilayer fully-connected NN (see Appendix C for an illustration of the NN structure). $\boldsymbol{X}_{Input}$ in the input layer includes all indices used to construct the index insurance contract, while I in the output layer represents the final payoff function solved by this system. In addition to an input layer and an output layer, there are $H(H \geq 1)$ additional hidden layers. The h-th $(h = 1, 2, \ldots, H)$ hidden layer $\boldsymbol{Z}^{(h)}$ contains p_h neurons, which are obtained by transforming the linear combination of the neurons from the previous layer through an activation function, f_h, elementwise. $\boldsymbol{\alpha}^{(h)}$ and $\boldsymbol{\omega}^{(h)}, h = 1, 2, \ldots, H$, are parameters of the linear combination. $\boldsymbol{\alpha}^{(h)}$ is a $(p_h \times 1)$-vector of bias units that captures the intercepts in the model; $\boldsymbol{\omega}^{(h)}$ is a $(p_h \times p_{h-1})$-dimensional weight matrix. $\boldsymbol{\alpha}^{(h)}$ and $\boldsymbol{\omega}^{(h)}$ $(h = 1, 2, \ldots, H)$ are learned through stochastic gradient descent (SGD), where their gradients are derived by backpropagation. This network is fully connected, in the sense that neurons between two adjacent layers are fully pairwise connected, but neurons within a layer have no connections. In summary, an NN structure is defined by its number of hidden layers H, the number of neurons in each hidden layer p_h, activation functions f_h, and the parameters $\boldsymbol{\alpha}^{(h)}, \boldsymbol{\omega}^{(h)}$ $(h = 1, 2, \ldots, H)$. A specific NN structure defines the feasible set $\mathcal{I}_0 \subset \mathcal{I}$ in the optimization problem (3).

Note that this framework conveniently guarantees that I is nonnegative, which is necessary for insurance indemnity payoffs, e.g., by making the last layer activation function, f_H, nonnegative. Throughout this paper, we use the rectified linear unit (RELU), defined by $f(x) = \max(x, 0)$, as the nonlinear activation function in our empirical analysis.

[5]Other machine learning alternatives could also be considered. For example, regression tree is a popular machine learning approach that could also address high-dimensionality and non-linearity. In Appendix O, we compare the performance of NN-based model with different tree-based models, including a simple regression tree, tree bagging, random forest, and tree boosting (see Gu et al. (2020) for more details). We find that the NN-based models perform better.

One might be concerned with the overfitting issue of neural networks. To avoid overfitting, we impose complexity constraints on function I by using early stopping. We also try other complexity constraints, including regularization methods (e.g., L^1 and L^2 regularization and their combination), elastic net, and the dropout method, and we find similar results.

3.2 Solving for the optimal policy using a penalty method

Two issues remain before solving for the optimal index insurance policy. First, the objective function in problem (3) is a utility function, not a typical loss function in machine learning. Second, there is a budget constraint in problem (3), that is, $P_L \leq \pi_e(I) \leq P_U$. The first issue can be solved by defining a customized loss function when we formulate an NN program.[6] To maneuver the second issue, we propose a penalty method.

Let's consider a sequence of unconstrained problems $\{\Phi_k\}_{k\geq 0}$:

$$\Phi_k = \min_{I\in\mathcal{I}_0} \left(-\frac{1}{n}\sum_{j=1}^{n} U\big(w - y_j + I(\boldsymbol{x}_j) - \pi_e(I)\big) + \phi_k \cdot g(I) \right), \tag{4}$$

where $\{\phi_k\}_{k\geq 0}$ is a sequence of increasing nonnegative real numbers tending toward infinity (i.e., $\phi_k \geq 0, \phi_{k+1} \geq \phi_k, \lim_{k\to\infty} \phi_k = +\infty$), and $g(I)$ is the penalty function. The penalty function is defined as a sum of squared errors (Luenberger et al., 1984), as follows:

$$g(I) = \left[\max\left(\pi_e(I) - P_U, 0\right)\right]^2 + \left[\max\left(P_L - \pi_e(I), 0\right)\right]^2. \tag{5}$$

Our index insurance design problem (3) then can be connected to the unconstrained problem (4), using the following theorem.

[6] In a standard NN-based statistical learning problem, the objective function is usually a loss function defined as certain distance measure (e.g., L^1 norm for mean absolute error, L^2 norm for mean squared error, the Huber loss, or the cosine similarity, all of which are commonly used built-in functions in Keras). For our problem, we create a customized loss function (4) using the backend functions in Keras.

Theorem 1 (Luenberger et al. (1984)). *Let I_k^* be a sequence of solutions solving the corresponding sequence of unconstrained problems $\Phi_k(I)$, as defined in problem* (4)*, where $\{\phi_k\}_{k\geq 0}$ is an increasing sequence tending toward infinity. Then any limit of $\{I_k^*\}_{k\geq 0}$ is a minimizer of the constrained problem* (3)*.*

Therefore, based on Theorem 1, we construct a sequence of increasing penalty coefficients, $\{\phi_k\}_{k\geq 0}$, and specify the penalty function, $g(I)$, according to Equation (5). The optimal contract for problem (3) can be solved from the unconstrained problem (4) iteratively. Appendix D summarizes the algorithm.

4 Performances of the NN-based index insurance

In this section, we evaluate the empirical performances of the proposed NN-based design of a weather index insurance policy. To proceed, we consider a representative corn farmer with a negative exponential utility function $U(w) = -\frac{1}{\alpha}e^{-\alpha w}$, where $\alpha > 0$ is the absolute risk aversion coefficient. Using a CARA utility provides some advantages. First, under this CARA utility, the optimal index insurance contract in problem (3) is independent of farmers' initial wealth level. This property is realistic from the practical viewpoint of designing insurance policies. It also makes results comparable among different policyholders. Second, the negative exponential utility function conveniently handles negative wealth (i.e., bankruptcy) cases, whereas other utility functions, such as power utility functions, may not be well defined when wealth is negative, which could occur under some catastrophic events. Therefore, we use a negative exponential utility function in our main results. As a robustness check, we consider other utility specifications, e.g., logarithmic and power utility functions, in Appendix L.5.

Section 4.1 summarizes the data used in this paper. In Section 4.2, we focus on the baseline case of a representative farmer when budget constraints are barely binding.[7] In other

[7]We will test the impact of budget constraints in the robustness analysis in Appendix L.3.

words, the farmer is primarily interested in maximizing her expected utility and accepts the equilibrium coverage level and its corresponding insurance premium. We study the reduction in basis risk and the utility improvement of the optimal policy in the baseline case. Section 4.3 explores the interpretability of the optimal policy. Finally, we compare the NN-based optimal policy with some simpler contracts in Section 4.4.

4.1 Data

4.1.1 Production loss and farmer's wealth data

We use a dataset consisting of county-level annual corn production experience for Illinois, obtained from the National Agricultural Statistics Service (NASS). Corn is the single most valuable agricultural commodity in the United States, and composes more than 45% of cropland by acreage in Illinois.[8] Another advantage of using Illinois corn data is its long history. We use a sample period from 1925 to 2018. To ensure stationarity of the loss experience over the long sample period, we first detrend the crop yields data with a second-order polynomial, estimated with a robust regression technique. Next, similar to Deng et al. (2007) and Harri et al. (2011), we adjust historical yields data to the 2018 level and other data heteroscedasticity. The detrending procedure also helps remove the impacts of long-run climate change or technology progress, such as improved genetics (cultivars), improved crop management, and other technological advances like the use of advanced farming equipment. As the long sample of corn yield data is only available at annual frequency, the number of historical yield observations is quite limited, especially compared with the high-dimensional weather-related covariates we want to utilize for each area. We circumvent this difficulty by assuming that crop yield losses are both time and space homogeneous after detrending.[9] This simplification expands the size of our data to 7,869 county-years (84 counties $\times$ 94 years, with 27 missing data points removed from the sample).

[8]Source: USDA's National Agricultural Statistics Service, Economic Research Service.

[9]We discuss the data homogeneity in Appendix G.

In the main results of this paper, we focus on hedging fluctuations in crop yields (i.e., production losses) and assume that the corn price is a constant of \$3.5 per bushel (\$/bu), which is the 5-year average of the inflation-adjusted corn price from 2014 to 2018.[10] That is, the dollar wealth is indexed on the most recent five-year price level. We further consider the price risk in Appendix M. The corn production loss data measured in bushels per acre (bu/acre) are then multiplied by the corn price to arrive at the monetary loss incurred by each farmer in dollars per acre.

The 2014-2018 USDA Agricultural Resource Management Survey and NASS show that net farm assets (i.e., farm assets minus farm debts) have a five-year average of \$456,977.[11] In addition, according to Illinois Farm Business Farm Management, the average size of Illinois grain farms is 1,176 acres.[12] To make the numerical results comparable and interpretable, we normalize all monetary quantities for the representative farmer by farm size. In other words, wealth levels, incomes, and other monetary quantities used in our analysis are all measured in dollars per acre, which is also consistent with the unit of our production loss data. Dividing the net farm assets by the farm size yields the initial wealth of $w_0 = \$389$/acre.

4.1.2 Climate and weather index data

We collect weather data from the PRISM Climate Group.[13] PRISM publishes monthly meteorological information on six climate variables, including precipitation, max/min temperatures, max/min vapor pressure deficit, and dew points, from 1895 to present for the conterminous United States at a 4-km resolution. We use all of the six monthly climate variables over 1925-2018. This gives a 72-dimensional weather index matrix for our optimal

[10]Specifically, we obtain the commodity prices for corn at harvest from the USDA Economic Research Service, which are estimated using data from USDA's Agricultural Resource Management Survey and other sources (see `https://www.ers.usda.gov/data-products/commodity-costs-and-returns/`). Inflation is adjusted for by the annual consumer price index (CPI) from the Bureau of Labor Statistics.

[11]Available at `https://www.nass.usda.gov/Surveys/Guide_to_NASS_Surveys/Ag_Resource_Management/`.

[12]See "Farm Income and Production Costs for 2017: Advance Report", available at `http://www.fbfm.org/pdfs/AdvanceReport17.pdf`.

[13]PRISM is the USDA's official climatological data, available at `http://prism.oregonstate.edu/`.

insurance policy design. Table 1 describes the weather variables from the PRISM data set. Appendix E provides summary statistics for the weather variables.

Table 1: **Weather indices.** This table summarizes the weather variables available from the PRISM data set. The sample period is 1925-2018.

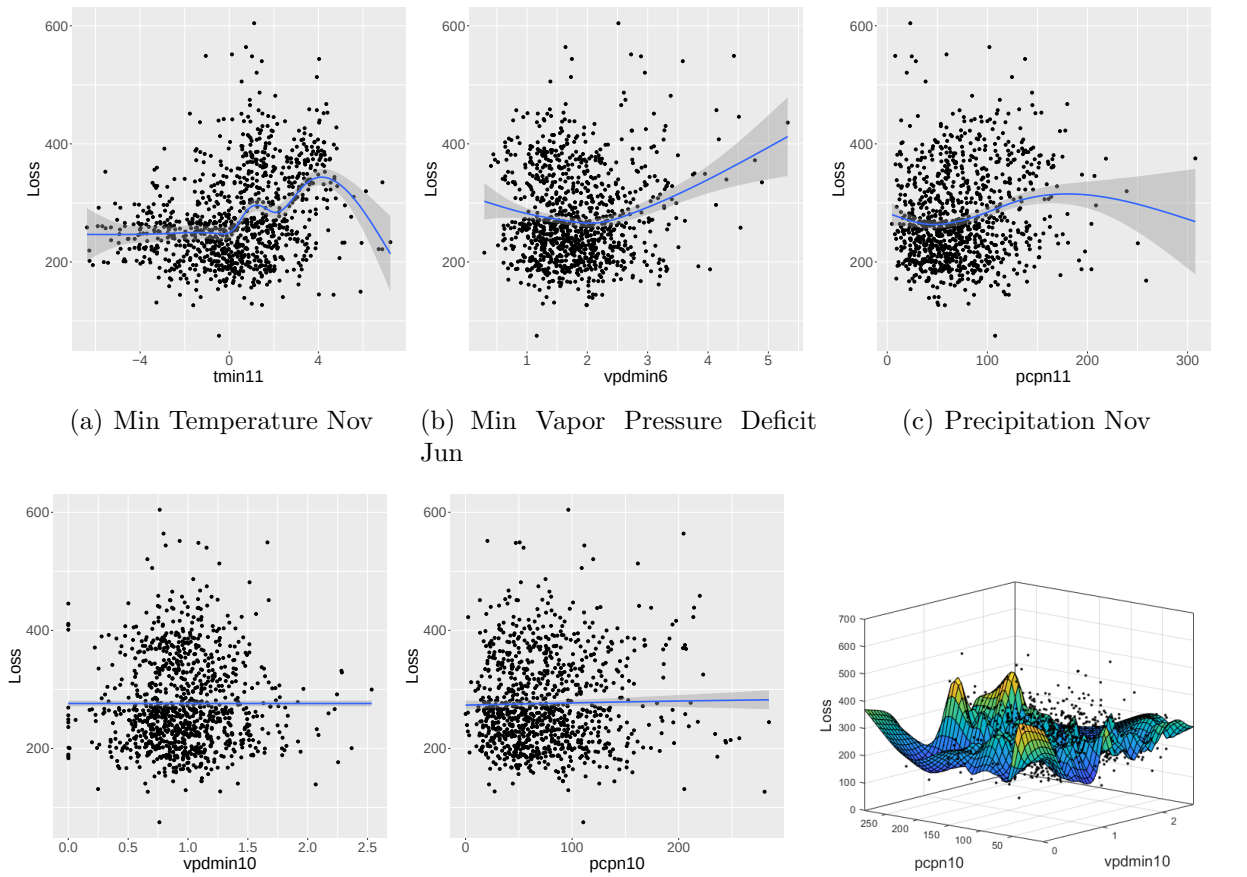
Variable	Description
pcpn$k$	Total precipitation (rain+melted snow) for month k (mm)
tmax$k$	Daily maximum temperature averaged over all days in month k (°C)
tmin$k$	Daily minimum temperature averaged over all days in month k (°C)
dpt$k$	Daily mean dew point temperature averaged over all days in month k (°C)
vpdmax$k$	Daily maximum vapor pressure deficit averaged over all days in month k (hPa)
vpdmin$k$	Daily minimum vapor pressure deficit averaged over all days in month k (hPa)
k	Calender month, i.e., $k = 1, 2, ..., 12$ for Jan - Dec

The impacts of weather indices on crop production losses can be nonmonotonic and highly nonlinear due to biological reasons (see, e.g., Schlenker and Roberts, 2009; Lobell et al., 2011; Rigden et al., 2020). Panels (a) - (c) of Figure 2 illustrate the nonlinear patterns of three selected weather indices (see Appendix F for scatterplots of all 72 weather indices). Furthermore, different weather indices might interact with each other and jointly affect production losses. Panels (d) and (e) of Figure 2 show that the min vapor pressure deficit in October and precipitation in October do not individually affect production losses. However, Panel (f) of Figure 2 shows that they jointly affect production losses, in a highly nonlinear way. These complexities cannot be captured by the linear models used by most current index insurance contracts, and, thus, more sophisticated contract design models are needed.

4.1.3 Insurance market data and the supply curve

We obtain insurance market data from the USDA National Summary of Business (SOB) Reports (1989-2017).[14] The variables that we collect include the total acres insured ($Acres$), premiums ($Prem$), liabilities ($Liab$), and indemnities ($Indem$). Liabilities stand for insurance guaranteed crop production levels. If the farmer's actual yields are lower than the liability level, the farmer will receive insurance payments. Indemnities correspond to real-

[14]Available at: `https://www.rma.usda.gov/en/Information-Tools/Summary-of-Business`.



(a) Min Temperature Nov

(b) Min Vapor Pressure Deficit Jun

(c) Precipitation Nov

(d) Min Vapor Pressure Deficit Oct

(e) Precipitation Oct

(f) Min Vapor Pressure Deficit Oct and Precipitation Oct

Figure 2: **Relationship between crop losses and weather indices.** Panels (a)-(e) visualize crop losses with five selected weather indices, based on 1,000 random draws from the sample. The blue curve is fitted by a generalized additive model. The shadow area indicates a 95% confidence interval. Panel (f) shows the joint effects of two weather indices used in Panels (d) and (e) on crop losses. The surface is fitted by a thin-plate smoothing spline model.

ized insurance payments. The USDA Risk Management Agency computes the loss cost ratio (LCR) as the ratio of indemnity over liability: $LCR = \frac{Indem}{Liab}$. The premium is then calculated as the expected LCR times the liability, multiplied by the loading parameter λ, that is, $Prem = \lambda \cdot \mathrm{E}(LCR) \cdot Liab$. Therefore, we can use this relationship to infer the realized λ every year from the market data. Insurance coverage per acre is calculated as $Cov = \frac{Indem}{Acres}$ (adjusted by inflation), representing the unit acre insurance supply at market equilibrium each year. We use the historical data to fit the supply curve as a power function of λ, using nonlinear least squares method. The fitted supply curve is $Cov = f_S(\lambda) = 7.04\lambda^{2.92} + 12.83$. Due to the data limitation, one might worry about the simultaneity bias of fitting the supply curve from historical data. We perform robustness checks by using alternative supply curves in Appendix L.1.

4.2 The optimal NN-based index insurance: The baseline case

We split the sample into three disjoint time periods, preserving the data temporal order, with the earliest 70% data as training set, then the next 15% as validation set, and the last 15% as test set. NN parameters are trained on training set for each given set of hyperparameters, including the number of neurons and hidden layers. Then the validation set is used to choose the optimal set of hyperparameters. Finally, we use the test set to evaluate the performance of the selected optimal NN-based index insurance. We adopt the ensemble method to improve the robustness of the analysis. Specifically, all models are averaged over 10 training runs.

We first consider the optimal index insurance whereby farmers are barely bound by any budget constraint. For example, P_L is set to zero and P_U is set to the farmer's total wealth. In other words, the farmer would like to pay any price that she could afford, to maximize her utility. We will discuss the effects of budget constraints in Appendix L.3. We set the absolute risk aversion, α, to 0.008, which corresponds to a relative risk aversion of 3.1.[15]

[15]This is in line with the literature. For example, Chavas and Holt (1990) find that maize and soybean

In Section 4.2.1, we discuss how to endogenously determine the loading parameter λ at equilibrium. Then we discuss the performance of the optimal index insurance of the baseline case in Section 4.2.3.

4.2.1 Determining the loading parameter at market equilibrium

We determine the endogenous loading parameter at market equilibrium via a reduced-form approach, as follows. First, we use the USDA SOB Reports market data to fit the supply curve, $Cov = f_S(\lambda)$, as described in Section 4.1.3. Second, for a given value of $\lambda \in [1.02, 1.5]$,[16] we obtain the corresponding optimal coverage of index insurance policies, using the NN-based approach. Given the pairs of λ and the optimal coverage, we fit the demand curve, $Cov = f_D(\lambda)$, using the piecewise cubic Hermite interpolating polynomial. Finally, we equate the supply and demand functions, $f_S(\lambda) = f_D(\lambda)$, to solve for the loading parameter at market equilibrium, λ^*. Therefore, farmers and insurers jointly determine the insurance premium in the equilibrium. Figure 3 illustrates an example of finding λ^* for the NN-based optimal index insurance with a 3-hidden-layer (64-64-16 neurons) structure, corresponding to the baseline model that we will discuss in Section 4.2.3. The equilibrium loading parameter is $\lambda^* = 1.2414$.

4.2.2 Selecting the optimal NN-based model

We consider and compare different neural network structures. For each neural network structure, we use the procedures described in Section 4.2.1 to compute the equilibrium loading parameter, λ^*. As we endogenously determine λ^*, it would be very computationally costly to endogenize the hyperparameters of the model. Instead, we examine a wide range of candidate models with different number of hidden layers and different number of neurons in each layer, and then select the optimal model based on the utility improvement in the validation set. Panel A of Table 2 shows the results without using index insurance. Panels

farmers have a relative risk aversion ranging from 1.41 to 7.62, using aggregated data.

[16] This interval covers the feasible loading parameter for index insurance in practice.

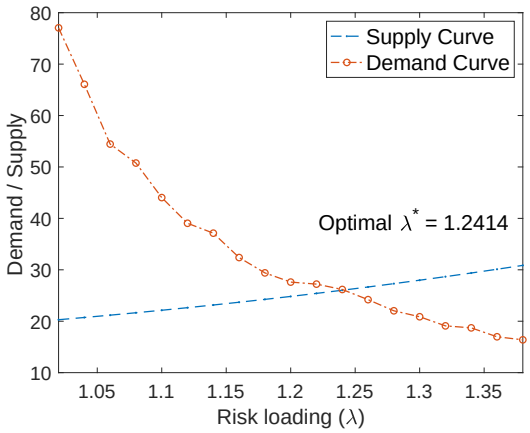


Figure 3: **The supply and demand curves of the index insurance.** The insurance supply curve is fitted from the USDA SOB Reports data with a power function. The demand curve is for the NN-based optimal index insurance with a 64-64-16 neurons structure, and is fitted with a piecewise cubic Hermite interpolating polynomial. The intersection gives the loading parameter $\lambda^* = 1.2414$ at market equilibrium.

B-E present the results from various NN-based index insurance contracts with different layer and neuron structures. For example, "$i-j-k$" indicates a three-hidden-layer NN model with i, j, and k neurons in the first, second, and third hidden layers, respectively. We compute the insurance premium and compare expected utilities with and without index insurance. For ease of interpretation, we also compute the CEW with and without index insurance.

Panel B shows that a simple NN with only one hidden layer and two neurons significantly improves farmers' expected utility by 6.54% and CEW by \$8.45/acre in the validation sample. In addition, adding more neurons or adding more hidden layers generally improves the model performances, as shown in Panels B-D. However, increasing the number of hidden layers from 3 to 4 (moving from Panel D to Panel E) does not necessarily improve the NN performance. We see that the three-hidden-layer NN with a 64-64-16 neurons structure yields the largest utility improvement and CEW improvement in the validation set. This NN-based contract provides a utility improvement of 10.60% in the training sample and 9.31% in the validation sample. We choose this as the baseline model (*NN72*, with a 64-64-16 neurons structure) and discuss its forecasting performance in the test set in Section 4.2.3.

Table 2: **Selecting an NN model**. This table summarizes the performances of several NN-based index insurance policies with various neuron structures. Panel A shows expected utility and certainty equivalent wealth (CEW) without index insurance, over the training and validation samples. Panels B - E present results with different neuron structures. For example, "$i-j-k$" indicates i, j, and k neurons in the first, second, and third hidden layers, respectively. Each panel reports the expected utility, percentage utility improvement, CEW, CEW improvement in dollars, CEW improvement in percentage, and insurance premium for each optimal insurance policy, for both training and validation samples. The risk loading parameter at equilibrium (λ^*) is reported in parentheses. The optimal NN structure is the one with the largest utility improvement in the validation set (i.e., the 64-64-16 neuron structure).

Panel A: Without insurance		
	Training	Validation
U w/o insurance	-3.99	-3.99
CEW w/o insurance	430.63	430.41

Panel B: 1-Hidden-Layer NN	2 ($\lambda^* = 1.1956$)		8 ($\lambda^* = 1.2231$)		64 ($\lambda^* = 1.2340$)	
	Training	Validation	Training	Validation	Training	Validation
U with insurance	-3.65	-3.73	-3.59	-3.71	-3.54	-3.65
U improvement	8.37%	6.54%	9.86%	7.18%	11.17%	8.69%
CEW with insurance	441.57	438.86	443.60	439.73	445.44	441.77
CEW improvement	10.93	8.45	12.97	9.32	14.81	11.36
CEW improvement (%)	2.54%	1.96%	3.01%	2.16%	3.44%	2.64%
Premium	29.81	22.48	30.82	22.93	32.07	29.06

Panel C: 2-Hidden-Layers NN	16-8 ($\lambda^* = 1.2336$)		64-8 ($\lambda^* = 1.2428$)		64-16 ($\lambda^* = 1.2351$)	
	Training	Validation	Training	Validation	Training	Validation
U with insurance	-3.58	-3.67	-3.58	-3.66	-3.57	-3.70
U improvement	10.16%	8.01%	10.31%	8.27%	10.50%	7.39%
CEW with insurance	444.02	440.85	444.23	441.20	444.50	440.00
CEW improvement	13.39	10.44	13.60	10.79	13.87	9.59
CEW improvement (%)	3.11%	2.43%	3.16%	2.51%	3.22%	2.23%
Premium	26.44	26.69	26.25	28.88	26.94	28.80

Panel D: 3-Hidden-Layers NN	64-16-8 ($\lambda^* = 1.2400$)		64-16-16 ($\lambda^* = 1.2428$)		**64-64-16 ($\boldsymbol{\lambda^* = 1.2414}$)**	
	Training	Validation	Training	Validation	**Training**	**Validation**
U with insurance	-3.57	-3.62	-3.56	-3.63	**-3.57**	**-3.62**
U improvement	10.53%	9.27%	10.64%	9.14%	**10.60%**	**9.31%**
CEW with insurance	444.54	442.56	444.70	442.40	**444.64**	**442.63**
CEW improvement	13.91	12.15	14.07	11.99	**14.00**	**12.22**
CEW improvement (%)	3.23%	2.82%	3.27%	2.78%	**3.25%**	**2.84%**
Premium	27.92	28.57	28.00	27.95	**28.44**	**28.99**

Panel E: 4-Hidden-Layers NN	64-16-8-8 ($\lambda^* = 1.2212$)		64-32-16-8 ($\lambda^* = 1.2445$)		64-64-16-16 ($\lambda^* = 1.2285$)	
	Training	Validation	Training	Validation	Training	Validation
U with insurance	-3.55	-3.63	-3.57	-3.68	-3.56	-3.68
U improvement	11.07%	9.09%	10.52%	7.98%	10.85%	7.78%
CEW with insurance	445.30	442.32	444.53	440.80	444.99	440.53
CEW improvement	14.67	11.92	13.90	10.39	14.35	10.12
CEW improvement (%)	3.41%	2.77%	3.23%	2.41%	3.33%	2.35%
Premium	29.32	30.07	28.91	27.04	27.63	26.77

4.2.3 Performances of the optimal index insurance

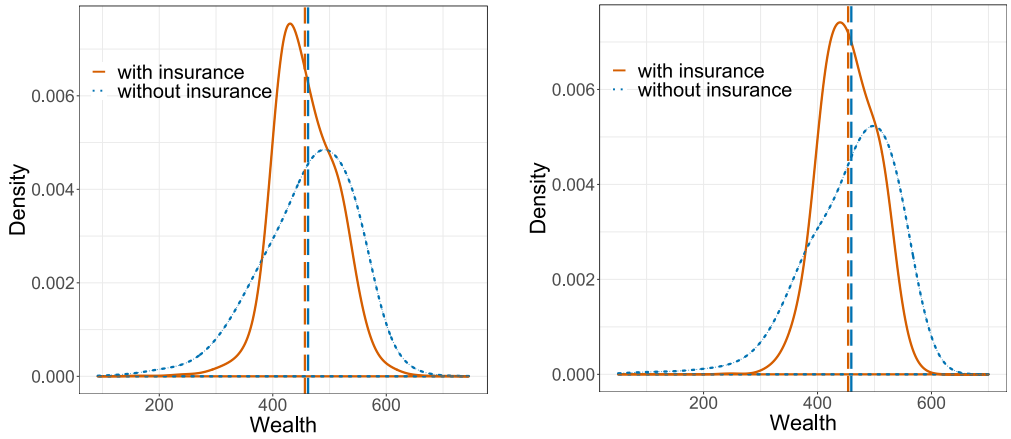
The performance of the optimal index insurance in the test sample is reported in Table 4. The baseline contract (*NN72*) provides a utility improvement of 14.35% in the test sample. With this insurance contract, the CEW improves by \$19.36/acre in the test set. Given an average farm size of 1,176 acres, the wealth improvement of the NN-based index insurance for a typical farm is economically significant, for example, \$22,767.36 in the test sample, which is about 4.98% of her wealth.

The optimal baseline index insurance policy has a premium of \$28.72 in the test sample. The corresponding equilibrium loading parameter is $\lambda^* = 1.2414$, implying a coverage level of \$23.13 in the test sample. Compared to the average insurance premium for Illinois corn farmers, which is around \$45.50/acre in 2017 (Smith, 2017), this optimal premium level is only 63% of the current premium level in practice. This lower premium for the proposed optimal index insurance would substantially increase the demand of index insurance in practice.

It is also shown that the proposed insurance achieves very similar expected utilities over training and test samples, suggesting that "overfitting" is of little concern. For example, the difference between policyholders' expected utilities with insurance in the training and test sample is less than 0.01, and the difference between CEW with insurance in the training and test sample is \$0.03, both of which are not sizable.

To further illustrate how the proposed NN-based insurance contract helps farmers improve their welfare, Figure 4 compares the wealth distributions between the "with insurance" case and the "without insurance" case. Without insurance, farmers' wealth distribution has a larger variation with a heavy left tail, i.e., the downside risk. In contrast, with the proposed index insurance, farmers' wealth distribution becomes less dispersed, and the left tail is significantly reduced.[17] We observe similar patterns in both training and test samples. Figure 4 demonstrates that our proposed index insurance policy effectively hedges the downside weather risk for farmers and substantially improves their utilities.

[17]The mean wealth is slightly lower with insurance, because of the insurance premium paid.



(a) Training sample (b) Test sample

Figure 4: **Farmers' wealth distribution: With index insurance versus without insurance.** The blue-dashed curve represents the probability density of wealth without using insurance, and the red-solid curve represents the probability density of wealth using the optimal NN-based index insurance. Panel (a) plots the training sample, and Panel (b) plots the test sample.

4.3 Interpreting the NN-based index insurance

Neural network results are often difficult to explain while economic interpretability is crucial for economics (Cong et al., 2020, 2021a). In this subsection, we perform a battery of sensitivity analyses to interpret the baseline case of NN-based index insurance contract.

4.3.1 Global interpretability

Following Cong et al. (2021a), we adopt a polynomial feature sensitivity analysis, which involves using the gradient-based characteristic importance method to examine contributions of feature inputs and extracts important features. Hence, it provides "economic distillation" of the NN-based index insurance, and increases interpretation of the "black box" insurance policy. This provides a global viewpoint since it is based on the whole sample. Given the optimal NN-based index insurance, $I(\boldsymbol{X})$, where $\boldsymbol{X}$ is the vector of weather indices, the sensitivity of $I(\boldsymbol{X})$ to the weather index X_k can be measured by its partial derivative with

respect to X_k:

$$\delta_{X_k}(\boldsymbol{X}) = \frac{\partial I(\boldsymbol{X})}{\partial X_k} = \lim_{\Delta x_k \to 0} \frac{I(\boldsymbol{X}) - I(X_k + \Delta x_k, \boldsymbol{X}_{-k})}{\Delta x_k}, \tag{6}$$

where $\boldsymbol{X}_{-k}$ is the vector of weather indices with the k^{th} index removed. Then the gradient-based sensitivity of insurance payoff with respect to X_k, $\mathcal{S}^g_{X_k}$, can be expressed as the average influence of index X_k to the optimal index insurance payoff, calculated as:

$$\mathcal{S}^g_{X_k} = \mathrm{E}\left(\delta_{X_k}\right) = \int_{\Omega} \delta_{X_k}(\boldsymbol{X}) d\mathbb{P}(\boldsymbol{X}), \tag{7}$$

where $\mathbb{P}(\boldsymbol{X})$ is the joint probability distribution of $\boldsymbol{X}$, Ω is the sample space, and the superscript g indicates the gradient-based sensitivity analysis. Empirically, $\mathcal{S}^g_{X_k}$ can be estimated by

$$\bar{\mathcal{S}}^g_{X_k} = \frac{1}{N} \sum_{i=1}^{N} \delta_{X_k}(\boldsymbol{x}_i), \tag{8}$$

where $\boldsymbol{x}_i, i = 1, \ldots, N$, are the sample data of $\boldsymbol{X}$ in the training set.[18]

We rank the importance of weather indices by the absolute value of $\bar{\mathcal{S}}^g_{X_k}$. Table 3 lists the five most important weather indices, as shown in the right panel.[19] For comparison, we also rank weather indices based on the absolute correlations between insurance payoffs and the indices, and the top five indices are shown in the left panel. We see that all five indices are different between these two panels, and most of them have very different ranks under these two ranking methods. While the correlations capture linear dependence, sensitivities from NN-based insurance provide a more general nonlinear dependence measure to study the relationship between insurance payoffs and weather indices. Those weather indices with a large absolute value of correlations are not necessarily the most important ones, whereas those variables with small correlations (tmin6, dpt6, dpt11, dpt12) may be very useful for the insurance. Finally, we notice that the five most important weather indices based on

[18] We find similar results over the test set.

[19] Appendix H ranks all indices by the gradient-based sensitivity analysis.

absolute correlations are all within the corn growing season, but the NN-based insurance contract picks up three weather indices outside the growing season, i.e., dew point temperature in November (dpt11), maximum temperature in December (tmax12), and dew point temperature in December (dpt12). In fact, agronomic studies show that hydrological conditions over nongrowing seasons might be important for corn production for two reasons. First, water accumulation in winter will affect the soil moisture that becomes available for corn in the next growing season.[20] Second, soil water accumulation during the nongrowing season affects cover crops, which are grown to improve soil quality and influence corn growth later.[21] This further illustrates the ability of neural networks to extract important nongrowing season information from the data, which is often overlooked by the linear functions in current practice.

Table 3: **Identifying important weather indices.** This table shows the 5 most important weather indices ranked by the absolute correlations between insurance payoffs and weather indices (left panel) or the gradient-based sensitivities of an NN-based insurance payoffs to the weather indices (right panel). Column "Rank Diff" shows the difference between two ranks for a weather index. See Table 1 for the index variable descriptions.

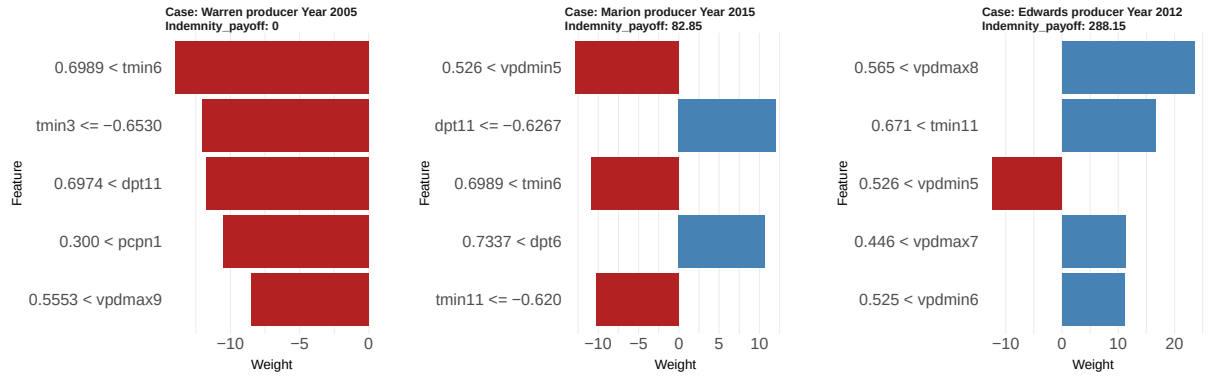
$Rank^{corr}$	Index	Absolute correlations	Rank Diff $Rank_g^{NN} - Rank^{corr}$	$Rank_g^{NN}$	Index	NN policy absolute $\bar{\mathcal{S}}^g$	Rank Diff $Rank_g^{NN} - Rank^{corr}$
1	**tmax8**	0.49	20	1	**tmin6**	10.30	-27
2	**tmax7**	0.46	11	2	**dpt6**	10.03	-37
3	**vpdmax8**	0.45	3	3	**dpt11**	9.58	-42
4	**tmin7**	0.41	27	4	**tmax12**	8.70	-3
5	**vpdmax7**	0.41	6	5	**dpt12**	7.76	-25

4.3.2 Local interpretability

From the marketing perspective, sometimes a local interpretation of the insurance contract may be more helpful, because a farmer often wants to know what weather variables

[20]Suyker and Verma (2008) find that cumulative evapotranspiration during the "nongrowing" seasons contributes to 20%–25% of the annual evapotranspiration totals for corn. Li et al. (2019) show that water storage over nongrowing seasons affects corn yields.

[21]Cover crops, such as cereal rye, which help build and improve soil fertility and quality, control diseases and pests, and promote biodiversity, are commonly integrated into corn production in Illinois. In fact, in Illinois, farmers who adopt cover crops may be eligible to receive an insurance premium discount in the following year through the Illinois Department of Agriculture (IDOA) Cover Crop Premium Discount Program (Illinois Department of Agriculture, 2020).



(a) Zero indemnity payoffs (b) Medium indemnity payoffs (c) Large indemnity payoffs

Figure 5: **Local explanation of the NN-based index insurance with LIME.** This figure displays three representative examples of local interpretable model-agnostic explanations (LIME) for the designed NN-based index insurance. Panels (a)-(c) show farmer-year pairs that receive indemnity payoffs of zero, medium, and large amounts, respectively. Five most important weather indices for each farmer-year pair are illustrated. Blue (Red) bars indicate that the corresponding indices contribute positively (negatively) to the indemnity payments, i.e., bad (good) weather conditions. Longer bars imply higher impacts that the indices have on indemnity payments.

will contribute to her own insurance payoff in a certain year. Therefore, we apply the Local Interpretable Model-agnostic Explanations (LIME, Ribeiro et al. (2016)) to provide local interpretations. LIME approximates the underlying black box model with a simpler and more interpretable local surrogate model, typically a linear regression model, to investigate a local region of the data. With LIME, farmers can understand the rationale behind the NN-based contract, assisting them to decide whether or not to trust the insurance payoffs.

Figure 5 displays some representative examples of LIME results. For example, for the farmer in Warren county, all the top five weather indices suggest a good harvest, and hence she would receive an insurance payoff of zero in 2005. By contrast, the farmer in Edwards county would receive a large insurance payoff of $ 288.15 per acre in 2012. The large payoff is due to a high vpdmax8, tmin11, vpdmax7, and vpdmin6, while a high vpdmin5 reduces the payoff. Such explanations will help improve the farmers' understanding and trust of the index insurance contract.

4.4 A comparison with other simple insurance contracts

Previously, we demonstrated the effectiveness of the NN-based index insurance design and assessed its interpretability. In this subsection, we further compare the NN-based insurance contract with other commonly used simple contracts, i.e., contracts with fewer indices and/or less nonlinear structures. This comparison highlights the benefits of using the NN-based insurance contract. In addition to the NN-based insurance contract, we consider some polynomial-based contracts, including (1) a linear insurance contract; (2) a quadratic insurance contract; and (3) a cubic insurance contract.[22] Depending on the contract structure, we use either all 72 weather indices if manageable or the most important one or five weather indices identified by gradient-based sensitivity analysis in the NN-based contract, discussed in Section 4.3.[23] Specifically, we consider the following seven index insurance contracts:

- *Linear contract with one input (Linear1)*: a linear contract written on a single temperature index, which is a currently used index insurance contract in practice;
- *Linear contract with five inputs (Linear5)*: only linear polynomials are used, with the top-five weather indices;
- *Quadratic contract with five inputs (Quadratic5)*: quadratic polynomials are used, with the top-five weather indices;
- *Cubic contract with five inputs (Cubic5)*: cubic polynomials are used, with the top-five weather indices;
- *NN with five inputs (NN5)*: NN with a 64-64-16 neuron structure, using the top-five weather indices;

[22]Polynomial terms might not be orthogonal, so using polynomials of a higher degree would introduce multicollinearity/robustness issues.

[23]Instead of using 72 weather indices, one might selectively use fewer weather indices and still achieve reasonable insurance contracts. Appendix J further compares NN-based models with different number of weather indices.

- *Linear contract with 72 inputs (Linear72)*: only linear polynomials are used, with all 72 weather indices;

- *Baseline model (NN72)*: NN with a 64-64-16 neuron structure, using all 72 weather indices. This is the baseline optimal contract discussed in Sections 4.2 and 4.3.

For each contract, the loading parameter is endogenously determined via the reduced-form approach described in Section 4.2.1. Table 4 presents the performances of these seven contracts in the test set. We compare the contract performances mainly by examining their utility improvements, CEW improvements, and risk reductions. We measure risk reductions by the standard deviation, or the 5% value-at-risk (VaR) of policyholders' wealth.

First, adding more weather indices as inputs significantly improves the contract performance of *Linear72*, relative to *Linear1* and *Linear5*. Second, comparing *Linear5*, *Quadratic5*, and *Cubic5*, we see that utility improves a little when adding nonlinear terms to the insurance contract in Panel A. We see similar improvements in CEW in Panel B, standard deviation of wealth in Panel D, and tail risk in Panel E. Third, using the same five weather indices, we see *NN5* outperforms *Linear5*, *Quadratic5*, and *Cubic5*. Fourth, the baseline model (*NN72*) delivers the best performance. *NN72* provides the highest expected utility and CEW to farmers, and it also achieves the highest risk reduction measured by standard deviation reduction and tail risk reduction. Furthermore, from the insurers' perspective, *NN72* is also the best contract because it provides the largest profits. Quantitatively, *Linear1* improves farmers' utility by 0.55%; *Linear72* improves their utility by 9.60%; *NN72* improves their utility by 14.35%. In terms of CEW, *Linear1*, *Linear72*, and *NN72* improve it by \$0.69, \$12.61, and \$19.36, respectively. Insurers profits are \$0.61/acre, \$5.02/acre, and \$5.59/acre, for the *Linear1*, *Linear72*, and *NN72*, respectively. This is due to higher market demand and hence a higher equilibrium loading, λ^*.

Basis risk often dissuades farmers from buying index insurance and makes insurances less cost effective. Policyholders are most concerned with the worst scenario when they suffer a huge loss in crop yields, but only receive little payments from the insurance. The opposite

Table 4: **Performances of the baseline insurance contract and alternative contracts** This table summarizes the performances of seven insurance contracts in the test sample, including (1) a linear insurance contract with one weather index (*Linear1*), which corresponds to a currently used contract; (2) a linear insurance contract with five weather indices (*Linear5*); (3) a quadratic insurance contract with five weather indices (*Quadratic5*); (4) a cubic insurance contract with five weather indices (*Cubic5*); (5) an NN-based contract with five weather indices (*NN5*); (6) a linear insurance contract with 72 weather indices (*Linear72*); and (7) the baseline model (*NN72*, an NN-based contract with 72 weather indices). Panel A summarizes expected utilities with and without (w/o) index insurance and the percentage of utility improvement. Panel B reports certainty equivalent wealth (CEW) with and without (w/o) index insurance and the CEW improvement in dollars and as a percentage. Panel C summarizes policy characteristics including policy premium, coverage, and profits of the insurer. Panel D summarizes the risk reduction effect of an index insurance policy, measured by the standard deviation of wealth. Panel E summarizes the tail risk reduction, measured by the 5%-level value-at-risk (VaR). The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Contract	Linear1 ($\lambda^* = 1.0255$)	Linear5 ($\lambda^* = 1.0730$)	Quadradic5 ($\lambda^* = 1.0785$)	Cubic5 ($\lambda^* = 1.0840$)	NN5 ($\lambda^* = 1.1778$)	Linear72 ($\lambda^* = 1.2133$)	NN72 (BL, $\lambda^* = 1.2414$)
Panel A: Utility improvement							
U with insurance	-4.14	-4.03	-4.03	-4.00	-3.78	-3.76	-3.57
U w/o insurance	-4.16	-4.16	-4.16	-4.16	-4.16	-4.16	-4.16
U improvement (%)	0.55%	3.08%	3.28%	3.84%	9.11%	9.60%	14.35%
Panel B: CEW improvement							
CEW with insurance	425.94	429.16	429.43	430.15	437.19	437.87	444.61
CEW w/o insurance	425.26	425.26	425.26	425.26	425.26	425.26	425.26
CEW improvement	0.69	3.91	4.17	4.90	11.94	12.61	19.36
CEW improvement (%)	0.16%	0.92%	0.98%	1.15%	2.81%	2.97%	4.55%
Panel C: Policy characteristics							
Premium	24.48	26.94	28.55	26.21	27.94	28.56	28.72
Coverage	23.87	25.10	26.47	24.18	23.72	23.54	23.13
Insurer Profit	0.61	1.83	2.08	2.03	4.22	5.02	5.59
Panel D: Risk reduction measured by standard deviation							
Std	75.13	71.05	70.01	69.69	62.86	59.36	47.49
Std w/o insurance	78.92	78.92	78.92	78.92	78.92	78.92	78.92
Std reduction	4.80%	9.97%	11.29%	11.69%	20.34%	24.78%	39.82%
Panel E: Risk reduction measured by Value-at-Risk (VaR)							
$\text{VaR}_{5\%}$	332.92	338.48	339.68	338.01	339.00	345.93	379.64
$\text{VaR}_{5\%}$ w/o insurance	325.91	325.91	325.91	325.91	325.91	325.91	325.91
$\text{VaR}_{5\%}$ improvement	7.02	12.58	13.77	12.10	13.09	20.03	53.73

case is also unappealing: although it does not hurt when farmers get a good harvest and receive some insurance payments at the same time, insurance companies will suffer losses in this case. This means that the insurance premiums are not effectively allocated to the worst scenarios when insurance payoffs are more needed. Appendix I compares the basis risk of these seven index insurance contracts. We see that except *NN72*, there is a notably large mismatch between losses and insurance payoffs, especially for the test set. Overall, our results demonstrate that, by reducing basis risk and solving for the endogenous insurance premium, the NN-based solution can significantly improve the demand of index insurance and increase the profit margin of insurers.

5 Robustness of NN-based index insurance

We investigate the robustness of the NN-based index insurance contract in several ways, using an NN structure of 64-64-16 neurons. First, we use data from three states adjacent to Illinois to perform the out-of-state tests. Second, we consider the impacts of over-insuring constraint. Third, we examine the impacts of weather predictability on insurance design. Appendix L provides additional robustness checks, including (1) the impacts of insurers' choices (various insurers' supply curves and exogenously given risk loading); (2) the impacts of farmers' characteristics (such as different coverage levels, different risk aversion levels, time-varying risk aversion, and alternative utility functions).

5.1 Out-of-state tests

As a more stringent out-of-sample test, we run some out-of-state tests for states adjacent to Illinois. Specifically, we use Illinois data as the training set to estimate the parameters in the NN-based index insurance policy and use data from the adjacent states as the test set to evaluate the model performance. We select three states adjacent to Illinois and that have similar latitudes: Indiana, Iowa, and Missouri, all of which are important corn

producers. Table 5 summarizes the results. We see that the NN-based index insurance performs reasonably well in these states. For example, it improves the expected utility by 8.69% in Indiana, 6.16% in Iowa, and 8.53% in Missouri. As for CEW, improvements are \$11.37/acre, \$7.95/acre, and \$11.15/acre in Indiana, Iowa, and Missouri, respectively. These results demonstrate the power of our proposed NN-based index insurance.

Table 5: **Out-of-state tests, using adjacent states.** We perform out-of-state tests for the NN-based index insurance. We use Illinois data as the training set to estimate the contract parameters. We use data from three states adjacent to Illinois, namely, Indiana, Iowa, and Missouri, as the test sets. The NN uses a 3-hidden-layer (64-64-16 neurons) structure. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance policies and the CEW improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurer. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes risk reduction at the tail, measured by the 5%-level value-at-risk (VaR).

Data	Training set	Test set		
States	Illinois	Indiana	Iowa	Missouri
Panel A: Utility improvement				
U with insurance	-3.55	-2.88	-3.95	-2.55
U w/o insurance	-4.02	-3.15	-4.21	-2.78
U improvement (%)	11.63%	8.69%	6.16%	8.53%
Panel B: CEW improvement				
CEW with insurance	445.21	471.48	431.77	486.74
CEW w/o insurance	429.75	460.11	423.82	475.59
CEW improvement	15.46	11.37	7.95	11.15
CEW improvement (%)	3.60%	2.47%	1.88%	2.34%
Panel C: Policy characteristics				
Premium	29.09	17.83	10.54	17.89
Coverage	23.43	14.36	8.49	14.41
Insurer Profit	5.66	3.47	2.05	3.48
Panel D: Risk reduction measured by standard deviation				
Std	51.29	51.17	53.39	64.20
Std w/o insurance	80.60	67.93	66.26	79.35
Std reduction	36.36%	24.66%	19.43%	19.10%
Panel E: Risk reduction measured by Value-at-Risk (VaR)				
$\text{VaR}_{5\%}$	381.12	396.90	354.55	394.52
$\text{VaR}_{5\%}$ w/o insurance	317.70	361.01	321.92	362.57
$\text{VaR}_{5\%}$ improvement	63.42	35.90	32.63	31.95

5.2 Over-insuring constraint

The over-insuring constraint usually is necessary for traditional optimal (re)insurance design:

$$I(\boldsymbol{X}) \leq \text{Loss}.$$

This constraint ensures that the indemnity payment from the insurance contract cannot exceed actual losses, which rules out the "over-insuring" issue. Indeed, previous studies find that there is an incentive for policyholders to over insure at the presence of basis risk (see, e.g., Doherty and Schlesinger, 1983). This constraint is less of a concern in our setting, for two reasons. First, there is little moral hazard and adverse selection in index insurance, because insurance payments cannot be manipulated by policyholders. Second, our baseline model exhibits very small basis risk and over-indemnifying payments. In this subsection, we further compare the numerical results for the optimal NN-based index insurance contract with and without the over-insuring constraint. The results are summarized in Table 6. We can see that the over-insuring constraint does not have significant impacts.

5.3 Weather predictability and conditional contracts

Our baseline model uses only current year weather indices as inputs. But one might wonder whether weather conditions are predictable which could affect the insurance contract design, because of the discrepancy between conditional and unconditional probability measures of weather indices. We address this concern in this subsection.

First, although short-term weather prediction technology has been improving dramatically over the last decades, long-term (e.g., several months or one-year ahead) weather is still unpredictable (Alley et al., 2019; Voosen, 2019). For example, Figure K.1 in Appendix K shows that currently the best forecast at the European Centre for Medium-Range Weather Forecasts runs out to around 10 days only. Given the unpredictable nature of annual weather

Table 6: **Over-insuring constraint.** We consider the optimal insurance performance with and without over-insuring constraint. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" indicates the baseline case studied in Section 4.2.

	With over-insuring constraint		Without over-insuring constraint (BL)	
Data	Training	Test	Training	Test
Panel A: Utility improvement				
U with insurance	-3.56	-3.57	-3.57	-3.57
U w/o insurance	-3.99	-4.16	-3.99	-4.16
U improvement (%)	10.61%	14.34%	10.60%	14.35%
Panel B: CEW improvement				
CEW with insurance	444.65	444.60	444.64	444.61
CEW w/o insurance	430.63	425.26	430.63	425.26
CEW improvement	14.02	19.34	14.00	19.36
CEW improvement (%)	3.25%	4.55%	3.25%	4.55%
Panel C: Policy characteristics				
Premium	28.37	28.54	28.44	28.72
Coverage	22.85	22.99	22.91	23.13
Insurer Profit	5.52	5.55	5.53	5.59
Panel D: Risk reduction measured by standard deviation				
Std	54.07	47.63	54.05	47.49
Std w/o insurance	81.94	78.92	81.94	78.92
Std reduction	34.02%	39.65%	34.04%	39.82%
Panel E: Risk reduction measured by Value-at-Risk (VaR)				
$\text{VaR}_{5\%}$	383.01	378.56	382.89	379.64
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	66.73	52.66	66.61	53.73

conditions, and the fact that index insurance contract is typically renewed annually, we expect the difference between the conditional and unconditional contracts to be small.

Second, to provide more direct evidence, in Table 7, we compare contracts that use the most recent one- (i.e., current), two-, and three-year weather conditions as inputs. Arguably, using longer historical weather indices as inputs can capture some weather predictability, if any, and those contracts should perform better. Such performance difference can signal the discrepancy among contracts conditioning on various weather information. However, we observe similar results from all these cases in Table 7. Therefore, the potential discrepancy between the conditional and unconditional contracts is likely negligible.

6 Extensions

In this section, we explore several extensions to the baseline model. First, we discuss the complexity issue associated with the NN-based insurance contract and propose potential ways to address it. Next, we extend our setting to corporate farming where the insureds are risk neutral. We consider an objective function of minimizing tail risk, instead of maximizing expected utility.

6.1 Contract complexity

In practice, contract complexity often deters insureds' participation in insurance markets, i.e., complexity aversion (Mador et al., 2000; Sonsino et al., 2002; Charness and Levin, 2005; Bernheim and Sprenger, 2020).[24] Admittedly, the NN-based contract is much more complicated than a simpler linear contract to farmers, even though the former provides much better protections. Therefore, it is important to alleviate farmers' concerns over contract complexity. First, we can increase the interpretability of the NN-based index insurance to improve farmers' understanding and trust in this product (see Section 4.3). The literature also finds

[24] In Appendix N, we consider the regulatory costs associated with contract complexity.

Table 7: **Contracts conditioning on various weather information.** This table compares contracts that use the most recent one- (i.e., current), two-, and three-year weather conditions as inputs. Panel A summarizes utilities with and without (w/o) index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance policies and CEW improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" indicates the baseline case studied in Section 4.2.

Weather conditions	Current year (BL) ($\lambda^* = 1.2413$)		Most recent 2 years ($\lambda^* = 1.2469$)		Most recent 3 years ($\lambda^* = 1.2456$)	
Data	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement						
U with insurance	-3.57	-3.57	-3.57	-3.59	-3.60	-3.58
U w/o insurance	-3.99	-4.16	-4.02	-4.15	-4.05	-4.15
U improvement (%)	10.60%	14.35%	11.19%	13.57%	11.12%	13.71%
Panel B: CEW improvement						
CEW with insurance	444.64	444.61	444.45	443.76	443.53	444.04
CEW w/o insurance	430.63	425.26	429.61	425.52	428.79	425.60
CEW improvement	14.00	19.36	14.84	18.24	14.73	18.44
CEW improvement (%)	3.25%	4.55%	3.45%	4.29%	3.44%	4.33%
Panel C: Policy characteristics						
Premium	28.44	28.72	28.96	28.28	28.81	28.53
Coverage	22.91	23.13	23.23	22.68	23.13	22.90
Insurer Profit	5.53	5.59	5.74	5.60	5.68	5.63
Panel D: Risk reduction measured by standard deviation						
Std	54.05	47.49	51.58	50.23	51.68	50.08
Std w/o insurance	81.94	78.92	81.17	78.97	81.07	79.14
Std reduction	34.04%	39.82%	36.46%	36.39%	36.25%	36.72%
Panel E: Risk reduction measured by Value-at-Risk (VaR)						
$\text{VaR}_{5\%}$	382.89	379.64	385.05	371.49	383.54	370.91
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	315.86	325.90	314.86	325.90
$\text{VaR}_{5\%}$ improvement	66.61	53.73	69.19	45.59	68.68	45.01

that education and insurance literacy can effectively improve insurance demand (Gaurav et al., 2011; Cai et al., 2020). Second, agricultural insurance often involves a public-private partnership (PPP). Government subsidies could be used to improve the communication and trust between the insurance companies and farmers. Last, we consider a hybrid index insurance that provides payoffs as the maximum of a linear contract and an NN-based contract. That is, this hybrid contract provides payoffs at least as good as a linear contract, which might help reduce the complexity concern of farmers.

6.1.1 Role of government subsidies

Government subsidies help launch insurance programs and improve insurance market participation. For example, Cai et al. (2020) show that government subsidies together with education programs enhance long-term adoption of index insurance programs. In U.S., the Federal Crop Insurance Program (FCIP) costs the federal government about $6.26 billion in 2019, with a subsidy ratio of about 60% (USDA, 2019; Rosa, 2018).

In this subsection, we quantitatively evaluate the impacts of government subsidies on the NN-based index insurance. Assume that the index insurance could be subsidized by the government for a proportion of θ. Then the terminal wealth of the farmer at the presence of the index insurance is $w_0 - Y + I(\boldsymbol{X}) - (1-\theta)\pi(I)$. The maximization problem is

$$\begin{cases} \min\limits_{I\in\mathcal{I}} & -\frac{1}{n}\sum\limits_{j=1}^{n} U\big(w_0 - y_j + I(\boldsymbol{x}_j) - (1-\theta)\pi_e(I)\big), \\ \text{s.t.} & P_L \le \pi_e(I) \le P_U. \end{cases} \tag{9}$$

We consider various subsidy rates, $\theta = \{0, 5\%, 10\%, 15\%, 19.45\%\}$, in Table 8. Note that $\theta = 19.45\%$ is the subsidy level such that farmers pay the actuarially fair premium and the government pays related costs and loadings, which is much lower than the rate of 60% in crop insurance practice (Rosa, 2018). With subsidies, the demand curve of policyholders moves upward, resulting in a higher loading parameter at equilibrium. Producer's utility, CEW, and coverages increase slightly. The insurer's profits increase significantly. This is

because the insurer has a small price elasticity, while farmers are more sensitive to prices. Therefore, subsidies cause larger changes to insurance demand curve. A higher profit margin will encourage more insurance companies to participate in the weather index insurance market. Therefore, government subsidies can significantly increase insurance participation and improve social welfare.

6.1.2 Hybrid contracts with guaranteed linear payoffs

In this subsection we propose a hybrid insurance contract as a temporary solution to the complexity concern. This hybrid insurance provides payoffs (I^{hybrid}) as the maximum of a linear contract and an NN-based contract, defined as follows:

$$I^{hybrid} = \max(\text{Linear}i, \text{NN}j) = \text{Linear}i + \max\left(0, \text{NN}j - \text{Linear}i\right),$$

where Linear$i$ and NNj denote generic payoff functions for a linear contract with i inputs and an NN-based index insurance with j inputs, respectively. With this hybrid contract, a farmer is guaranteed with a linear payoff which she could well understand and trust, and meanwhile enjoys the upside potential from the NN-based contract. That is, this hybrid insurance can simultaneously benefit from easy interpretability of a linear contract and large basis risk reduction of the NN-based contract.

We consider three hybrid contracts where an NN-based contract (*NN1*, *NN5*, or *NN72*) is combined with *Linear1* (the simplest linear contract). Table 9 summarizes the performances of these three hybrid contracts in the test sample. We can see that the hybrid contracts perform very close to the corresponding NN-based contracts. For example, the utility improvement of max(*Linear1*, *NN72*) is 14.02%, which is only slightly lower than that of the *NN72* contract (14.35%). Also note that the hybrid contracts generally have higher premiums due to higher indemnity payments. Although hypothetical, such analysis provides important and direct evidence that our proposed NN-based index insurance is very effective

Table 8: **Impacts of government subsidies.** We consider an NN-based index insurance with various government subsidy rates, i.e., $\theta = 0, 5\%, 10\%, 15\%$, or 19.45%. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance policies and the CEW improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage and profits for the insurer. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" represents the baseline case studied in Section 4.2. The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Subsidy rate	$\theta = 0$ (BL) ($\lambda^* = 1.2413$)		$\theta = 0.05$ ($\lambda^* = 1.2948$)		$\theta = 0.1$ ($\lambda^* = 1.3359$)		$\theta = 0.15$ ($\lambda^* = 1.3911$)		$\theta = 0.1945$ ($\lambda^* = 1.4400$)	
Data	Training	Test	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement										
U with insurance	-3.57	-3.57	-3.56	-3.56	-3.54	-3.54	-3.52	-3.52	-3.51	-3.50
U w/o insurance	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16
U improvement (%)	10.60%	14.35%	10.79%	14.52%	11.26%	15.01%	11.62%	15.41%	12.00%	15.84%
Panel B: CEW improvement										
CEW with insurance	444.64	444.61	444.91	444.87	445.56	445.59	446.07	446.17	446.61	446.82
CEW w/o insurance	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26
CEW improvement	14.00	19.36	14.28	19.61	14.93	20.33	15.43	20.91	15.97	21.56
CEW improvement (%)	3.25%	4.55%	3.32%	4.61%	3.47%	4.78%	3.58%	4.92%	3.71%	5.07%
Panel C: Policy characteristics										
Premium	28.44	28.72	29.90	29.79	32.77	33.09	36.36	36.29	37.11	39.54
Coverage	22.91	23.13	23.09	23.01	24.53	24.77	26.14	26.09	25.77	27.46
Insurer Profit	5.53	5.59	6.81	6.78	8.24	8.32	10.22	10.20	11.34	12.08
Panel D: Risk reduction measured by standard deviation										
Std	54.05	47.49	53.91	47.58	53.02	46.34	52.10	45.44	52.36	44.56
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	34.04%	39.82%	34.22%	39.71%	35.30%	41.28%	36.42%	42.42%	36.10%	43.54%
Panel E: Risk reduction measured by Value-at-Risk (VaR)										
$\text{VaR}_{5\%}$	382.89	379.64	382.92	378.89	383.70	380.40	384.96	384.57	385.29	386.77
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	66.61	53.73	66.64	52.99	67.42	54.50	68.69	58.67	69.02	60.86

in providing protections to farmers, even after considering complexity aversion. Moreover, this analysis also sheds light on the implementation of the NN-based index insurance in practice. Initially, insurance companies could collaborate with the government and provide some subsidized programs of such hybrid index insurance contracts, together with certain types of insurance education programs, so as to gain trust from farmers and improve insurance demand. Once the idea of this hybrid insurance program becomes well accepted by farmers, the hybrid contract may be gradually phased into the cheaper and more efficient NN-based index insurance.

6.2 Corporate farming

The framework discussed in this paper can be easily extended to corporate farming or internal risk management of insurance companies. Corporate farms usually are risk neutral and interested in reducing tail risk to achieve solvency in extreme scenarios. Therefore, policyholders aim to minimize their tail risk instead of maximizing expected utility. In particular, we consider the following tail risk minimizing problem:

$$\min_{I\in\mathcal{I}} -\mathrm{VaR}_{5\%}\left(\{w_0 - y_j + I(\boldsymbol{x}_j) - \pi_e(I)\}_{j=1,\dots,n}\right), \tag{10}$$

where $\mathrm{VaR}_{5\%}$ is the 5%-level Value-at-Risk (VaR) which is a commonly used tail risk measure. Table 10 summarizes the results. We see that, compared to the utility maximization baseline framework, the improvements in $\mathrm{VaR}_{5\%}$ are larger when farmer maximizes $\mathrm{VaR}_{5\%}$, while the improvements in expected utility and CEW are slightly lower. It is also shown that the designed contract is substantially more expensive than the baseline contract, indicating that it is more costly to manage tail risk. But such premium level is still affordable in the case of corporate farming.

Table 9: **Performances of the linear contract, NN-based contracts and hybrid contracts.** This table compares the performances of three hybrid contracts in the test sample, including a contract with the maximum payment between *Linear1* and *NN1*, a contract with the maximum payment between *Linear1* and *NN5*, and a contract with the maximum payment between *Linear1* and *NN72*. For ease of comparison, we also list the results of *Linear1*, *NN1*, *NN5*, and *NN72*. Panel A summarizes expected utilities with and without (w/o) index insurance and the percentage of utility improvement. Panel B reports certainty equivalent wealth (CEW) with and without (w/o) index insurance and the CEW improvement in dollars and as a percentage. Panel C summarizes policy characteristics including policy premium, coverage, and profits of the insurer. Panel D summarizes the risk reduction effect of an index insurance policy, measured by the standard deviation of wealth. Panel E summarizes the tail risk reduction, measured by the 5%-level value-at-risk (VaR). The risk loading parameter (λ^*) for each contract is reported in parentheses.

Contract	Linear1 ($\lambda^* = 1.0255$)	NN1 ($\lambda^* = 1.0512$)	NN5 ($\lambda^* = 1.1778$)	NN72 (BL, $\lambda^* = 1.2414$)	max($Linear1, NN1$) ($\lambda^* = 1.0353$)	max($Linear1, NN5$) ($\lambda^* = 1.0899$)	max($Linear1, NN72$) ($\lambda^* = 1.1114$)
Panel A: Utility improvement							
U with insurance	-4.14	-4.14	-3.78	-3.57	-4.14	-3.74	-3.58
U w/o insurance	-4.16	-4.16	-4.16	-4.16	-4.16	-4.16	-4.16
U improvement (%)	0.55%	0.47%	9.11%	14.35%	0.47%	10.18%	14.02%
Panel B: CEW improvement							
CEW with insurance	425.94	425.84	437.19	444.61	425.84	438.68	444.14
CEW w/o insurance	425.26	425.26	425.26	425.26	425.26	425.26	425.26
CEW improvement	0.69	0.58	11.94	19.36	0.59	13.42	18.88
CEW improvement (%)	0.16%	0.14%	2.81%	4.55%	0.14%	3.16%	4.44%
Panel C: Policy characteristics							
Premium	24.48	27.24	27.94	28.72	30.90	40.24	41.87
Coverage	23.87	25.91	23.72	23.13	29.84	36.92	37.67
Insurer Profit	0.61	1.33	4.22	5.59	1.05	3.32	4.20
Panel D: Risk reduction measured by standard deviation							
Std	75.13	72.92	62.86	47.49	73.07	61.84	52.06
Std w/o insurance	78.92	78.92	78.92	78.92	78.92	78.92	78.92
Std reduction	4.80%	7.60%	20.34%	39.82%	7.41%	21.64%	34.03%
Panel E: Risk reduction measured by Value-at-Risk (VaR)							
$\text{VaR}_{5\%}$	332.92	335.30	339.00	379.64	334.96	346.23	371.79
$\text{VaR}_{5\%}$ w/o insurance	325.91	325.91	325.91	325.91	325.91	325.91	325.91
$\text{VaR}_{5\%}$ improvement	7.02	9.39	13.09	53.73	9.05	20.32	45.88

Table 10: **Tail risk optimization.** We consider a risk-neutral agent with a tail risk minimization objective. The tail risk measure considered is $\text{VaR}_{5\%}$. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance policies and the CEW improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurer. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes risk reduction at the tail, measured by the 5%-level value-at-risk (VaR).

Objective function	$\text{VaR}_{5\%}$ maximization		Utility maximization (BL)	
Data	Training	Test	Training	Test
Panel A: Utility improvement				
U with insurance	-3.58	-3.60	-3.57	-3.57
U w/o insurance	-3.99	-4.16	-3.99	-4.16
U improvement (%)	10.19%	13.43%	10.60%	14.35%
Panel B: CEW improvement				
CEW with insurance	444.07	443.28	444.64	444.61
CEW w/o insurance	430.63	425.26	430.63	425.26
CEW improvement	13.44	18.02	14.00	19.36
CEW improvement (%)	3.12%	4.24%	3.25%	4.55%
Panel C: Policy characteristics				
Premium	46.37	45.17	28.44	28.72
Coverage	37.36	36.39	22.91	23.13
Insurer Profit	9.02	8.79	5.53	5.59
Panel D: Risk reduction measured by standard deviation				
Std	51.65	43.45	54.05	47.49
Std w/o insurance	81.94	78.92	81.94	78.92
Std reduction	36.97%	44.94%	34.04%	39.82%
Panel E: Risk reduction measured by Value-at-Risk (VaR)				
$\text{VaR}_{5\%}$	388.57	386.91	382.89	379.64
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	72.30	61.01	66.61	53.73

7 Conclusion

Index insurance could effectively manage systemic weather risk. However, the current insurance has large basis risk and is less cost effective, which leads to low insurance demand. In this paper, we formulate a neural network-based design of an index insurance contract which helps to reduce basis risk. Moreover, we consider endogenous insurance premium and demand formation to further improve the cost effectiveness of index insurance.

We illustrate the superior performance of this framework by applying it to corn farmers in Illinois. Results show that the NN-based index insurance contract effectively reduces basis risk and greatly outperforms other contracts (e.g., piecewise linear contract, quadratic contract, or cubic contract). This NN-based insurance contract significantly increases market demand, mitigates downside risk, and improves farmers' utility, even when we consider some frictions in practice (e.g., over-insuring constraints, regulatory costs, and contract complexity aversion). Our framework can be easily extended to other settings (e.g., the revenue index insurance and corporate farming). Overall, our results suggest the promise of using the NN-based model to design a broad class of financial products. Nevertheless, the marketing and operating perspectives of NN-based insurance contracts need further investigations, which we leave for future work.

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Online Appendix

A An example of an overfitted solution to problem (3)

Let's consider a special case of problem (3) as an illustrating example. Let $\{(\boldsymbol{x}_j, y_j)\}_{j=1,...,n}$ be a realized sample of $(\boldsymbol{X}, Y)$. Consider the minimization problem:

$$\begin{cases} \min\limits_{I\in\mathcal{I}} & -\dfrac{1}{n}\sum\limits_{j=1}^{n} U[w - y_j + I(\boldsymbol{x}_j) - \pi_e(I)] \\ \text{s.t.} & P_L \leq \pi_e(I) = \dfrac{\lambda}{n}\sum\limits_{j=1}^{n} I(\boldsymbol{x}_j) \leq P_U, \end{cases} \tag{A.1}$$

where $\mathcal{I} := \{I : \mathbb{R}^p \mapsto \mathbb{R}^+ | I \text{ is measurable}\}$. For simplicity we also replace the budget constraint by:

$$P_L = P_U = P = \lambda\frac{1}{n}\sum_{j=1}^{n} I(\boldsymbol{x}_j).$$

Then we have the following proposition:

Proposition 1 (Jensen's inequality)**.** *For any concave utility function U and any deterministic function I such that $P = \lambda\frac{1}{n}\sum_{j=1}^{n} I(\boldsymbol{x}_j)$, we have:*

$$\begin{aligned} \frac{1}{n}\sum_{j=1}^{n} U[w - y_j + I(\boldsymbol{x}_j) - P] &\leq U\left[\frac{1}{n}\sum_{j=1}^{n}\{w - y_j + I(\boldsymbol{x}_j) - P\}\right] \\ &= U\left[w - \frac{1}{n}\sum_{j=1}^{n} y_j + \frac{P}{\lambda} - P\right], \end{aligned}$$

with equality if and only if $I(\boldsymbol{x}_j) - y_j$ is a constant. Therefore the optimal solution I^* is given by,

$$I^*(\boldsymbol{x}) = \begin{cases} y_j + P/\lambda - \frac{1}{n}\sum\limits_{j=1}^{n} y_j, \ \textit{if } \boldsymbol{x} = \boldsymbol{x}_j, j = 1, 2, ..., n, \\ \textit{any arbitrary number, otherwise.} \end{cases} \tag{A.2}$$

This solution is "overfitted": although it mathematically optimizes problem (A.1), it says nothing about what the amount of indemnity should be for a new data sample. In fact, the problem comes from the fact that the admissible functional space $\mathcal{I}$ is too large and contains functions that are not constrained, or smooth enough. On the other hand, for instance, we constrain the space $\mathcal{I}$ to be the space of linear functions, then equations (A.2) cannot be satisfied for all indices $j = 1, ..., n$. Such a solution is too smooth, and usually result in a poor fit. Thus the challenge is to find a trade-off between these two extreme cases, that is, to propose suitable functional constraints on the $\mathcal{I}$.

B Feasible sets

We want to choose a feasible set, $\mathcal{I}_0$, which balances between flexibility and stability. $\mathcal{I}_0$ should be large enough to include candidate payoff functions that capture intricate (nonlinear, nonmonotonic) relationships between the high-dimensional indices and losses, yet $\mathcal{I}_0$ should also exclude "ill-behaved" ones in $\mathcal{I}$, which are sensitive to the sample data and cannot be an appropriate insurance contract. Such trade-off is illustrated in Figure B.1. The blue star in $\mathcal{I}$ is the global optimal contract, which may not be obtained.[25] The red triangle illustrates a highly unstable, overfitted contract, which we want to avoid. The dotted-grey circle area, $\tilde{\mathcal{I}}_0 \subset \mathcal{I}$, is a set of piecewise linear contracts. Although quite stable, $\tilde{\mathcal{I}}_0$ is far away from the blue star due to its restrictive functional form. Our goal is to expand the boundary of the feasible set towards $\mathcal{I}_0$, and obtain the optimal contract that falls within the intersection area, which is represented by the blue diamond. This optimal contract sacrifices a little stability but achieves much more flexibility and hence a large amount of basis risk reduction.

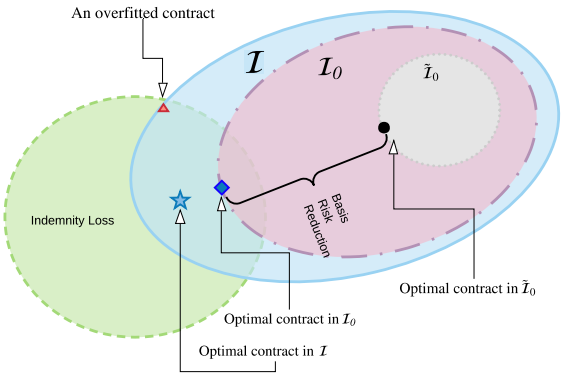


Figure B.1: **Feasible sets and optimal contracts.** This figure compares three different feasible sets and their corresponding optimal contracts. The dashed-green circle area represents the indemnity loss, which is the actual loss experienced by the policyholder. The general feasible set, $\mathcal{I}$, is represented by the solid-blue circle area and the blue star denotes the global optimal contract. The dotted-grey circle area, $\tilde{\mathcal{I}}_0$, is a feasible set of all piecewise linear contracts. The black dot at the edge of $\tilde{\mathcal{I}}_0$ is the optimal piecewise linear contract, i.e., the contract with the smallest basis risk within $\tilde{\mathcal{I}}_0$. The dotted-blue area, $\mathcal{I}_0$, represents the feasible set we explore. Its optimal contract is denoted by the blue diamond. The red triangle illustrates an overfitted contract.

[25] This is due to the fact that we replace the expectation in problem (1) with its empirical counterpart.

C Neural network structure

Figure C.1 illustrates a neural network with H-hidden layers.

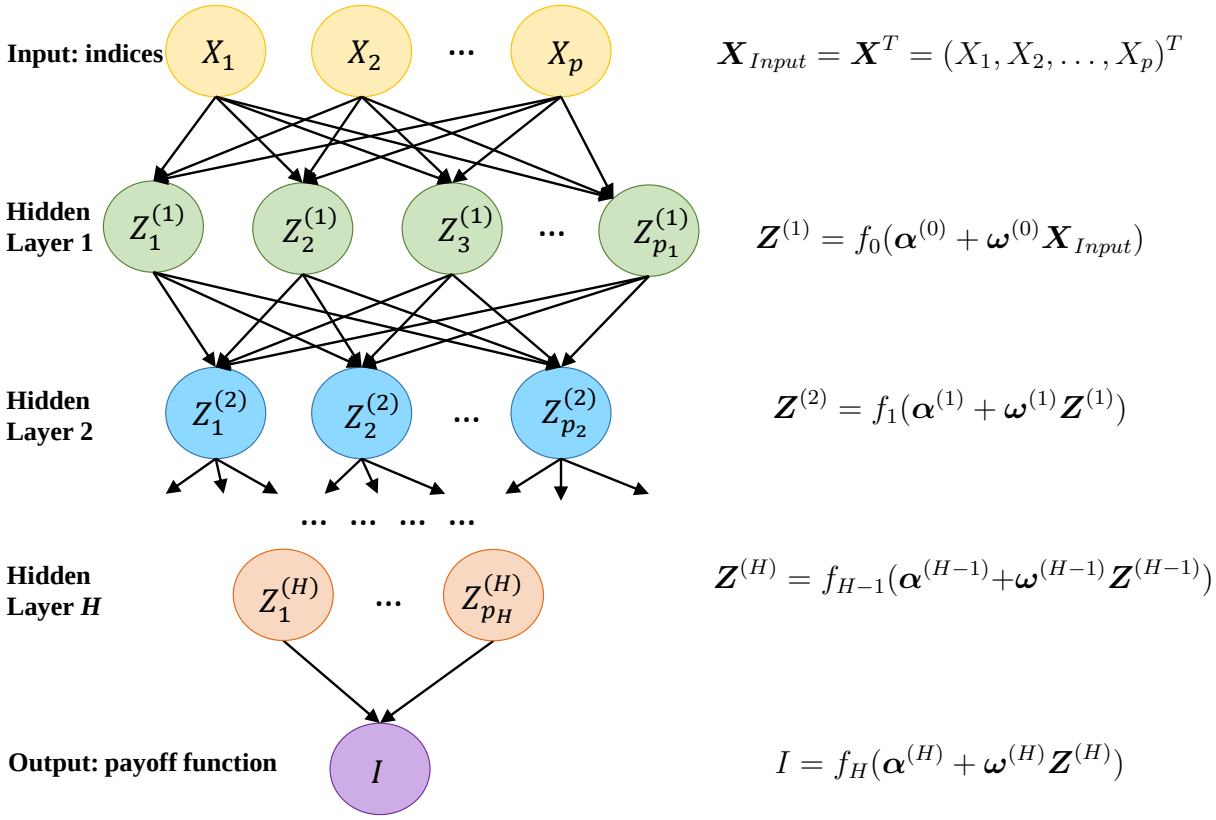


Figure C.1: **An illustration of neural networks with H-hidden layers.** This is an example of the fully-connected architecture in which neurons between two adjacent layers are fully pairwise connected, but neurons within a layer have no connections. f_h is an activation function; $\boldsymbol{\alpha}^{(h)}$ and $\boldsymbol{\omega}^{(h)}$ are parameters of the linear combination, $h = 1, 2, \ldots, H$.

D Algorithm: Solve for the optimal index insurance policy

Output: An optimal index insurance policy I
Input : Index data $\boldsymbol{X}$ and loss data Y
Build and initialize a neural network;
Initialization: $k = 0, \phi_0 = \epsilon_1$, obtain I by solving an unconstrained problem Φ_0;
while $|I - I_{last}| > \epsilon_3$ **or** $g(I) > \epsilon_2$ **or** $|\pi_e(I) - \pi_e(I_{last})| > \epsilon_4$ **do**
    Set $I_{last} = I$, $\pi_e(I_{last}) = \pi_e(I)$;
    Update $k \leftarrow k + 1$;
    Train the neural network and obtain the optimal I for problem $\Phi_k(I)$:

$$\Phi_k(I) = -\frac{1}{n}\sum_{j=1}^{n} U\big(w - y_j + I(\boldsymbol{x}_j) - \pi_e(I)\big) + \phi_k \cdot g(I),$$

    where the loss function is customized according to $\Phi_k(I)$ and I_{last} is set to the initial value of optimization;
    Update $g(I)$ and $\pi_e(I)$;
end
return (I)

Algorithm 1: Solve for the optimal index insurance policy.

E Data Summary

Tables E.1 and E.2 show summary statistics of the 72 weather indices used for empirical analysis. Statistics including mean, standard deviation, minimum, 25^{th} and 75^{th} percentiles, and maximum, are presented. The sample period is 1925-2018.

Table E.1: Summary statistics of weather indices. See descriptions for index variables abbreviations in Table 1.

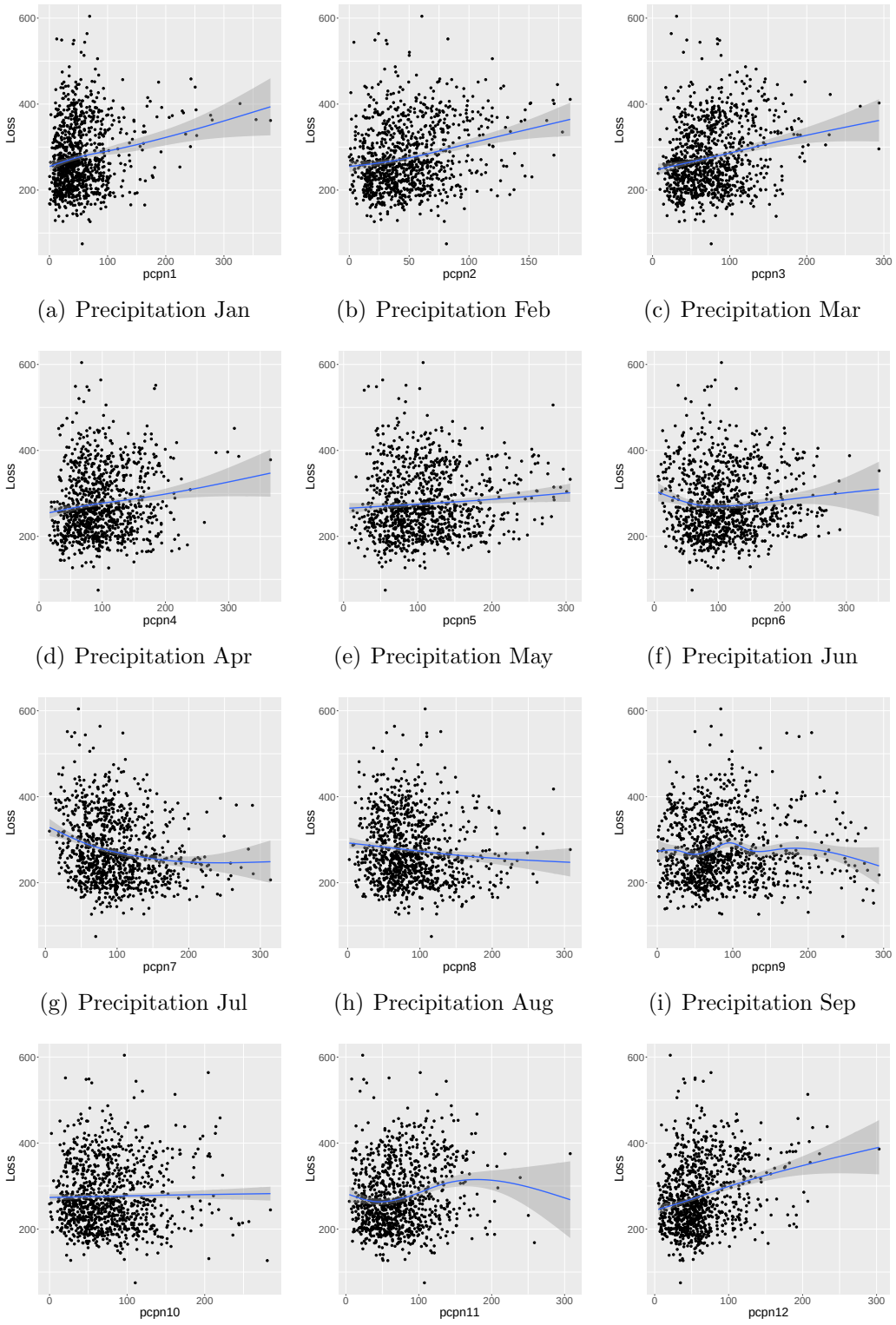
	pcpn1	pcpn2	pcpn3	pcpn4	pcpn5	pcpn6	pcpn7	pcpn8	pcpn9	pcpn10	pcpn11	pcpn12
Mean	53.36	49.69	77.37	98.32	106.95	108.91	94.97	87.04	86.25	74.84	74.97	62.08
StD	41.73	31.35	42.99	47.51	54.25	52.62	47.61	45.58	55.35	45.60	44.94	39.53
Min	0.03	0.06	6.86	16.60	7.71	4.28	3.97	2.30	0.21	0.02	4.28	3.90
Q_{25}	26.11	27.27	45.38	63.49	67.21	71.22	61.45	55.14	47.90	42.48	42.02	35.36
Q_{75}	68.76	65.37	99.57	125.26	133.62	138.28	120.13	109.56	111.79	98.32	100.17	78.26
Max	411.75	260.57	372.58	413.99	365.64	419.15	347.60	371.80	339.07	310.42	322.15	338.16
	dpt1	dpt2	dpt3	dpt4	dpt5	dpt6	dpt7	dpt8	dpt9	dpt10	dpt11	dpt12
Mean	-7.66	-5.91	-1.57	3.64	9.58	14.97	17.61	17.02	12.60	6.00	0.06	-5.13
StD	3.50	3.36	2.72	2.31	2.43	1.81	1.47	1.51	1.87	2.20	2.28	3.18
Min	-20.75	-17.26	-10.10	-3.96	2.64	9.19	12.93	12.49	7.32	-3.59	-8.08	-16.12
Q_{25}	-9.89	-8.19	-3.52	2.03	7.89	13.76	16.55	16.01	11.33	4.70	-1.42	-7.16
Q_{75}	-5.26	-3.46	0.20	5.27	11.26	16.25	18.62	18.08	13.90	7.39	1.69	-2.89
Max	2.11	2.14	7.22	10.09	16.84	20.35	22.35	21.52	18.51	12.57	6.71	4.80
	tmax1	tmax2	tmax3	tmax4	tmax5	tmax6	tmax7	tmax8	tmax9	tmax10	tmax11	tmax12
Mean	1.73	4.24	10.37	17.75	23.59	28.58	30.71	29.74	26.27	19.68	11.11	3.88
StD	3.77	3.87	3.60	2.60	2.36	2.02	2.00	1.99	2.17	2.54	2.86	3.39
Min	-9.45	-8.17	-0.42	9.11	16.72	23.08	24.23	24.04	18.94	9.35	2.53	-7.20
Q_{25}	-0.82	1.58	7.82	15.93	21.90	27.17	29.40	28.39	24.68	18.07	9.16	1.67
Q_{75}	4.42	7.02	12.84	19.62	25.25	29.85	31.89	30.99	27.79	21.19	13.13	6.27
Max	12.51	15.14	21.65	25.03	30.58	35.78	38.13	37.82	33.48	28.53	18.97	13.41

Table E.2: Summary statistics of weather indices (cont'd). See descriptions for index variables abbreviations in Table 1.

	tmin1	tmin2	tmin3	tmin4	tmin5	tmin6	tmin7	tmin8	tmin9	tmin10	tmin11	tmin12
Mean	-7.84	-5.88	-0.77	5.22	10.89	16.11	18.22	17.10	12.86	6.56	0.45	-5.23
StD	3.81	3.71	2.92	2.22	2.17	1.74	1.56	1.75	1.89	2.02	2.30	3.40
Min	-21.01	-19.34	-12.21	-2.61	4.93	10.41	13.32	11.29	7.58	-0.48	-7.93	-17.39
Q_{25}	-10.30	-8.28	-2.73	3.68	9.33	14.94	17.15	15.95	11.53	5.27	-1.01	-7.32
Q_{75}	-5.13	-3.16	1.14	6.80	12.38	17.32	19.30	18.27	14.15	7.92	2.03	-2.89
Max	1.70	3.34	9.21	11.92	18.07	22.11	23.33	24.05	19.71	13.04	8.15	4.00
	vpdmax1	vpdmax2	vpdmax3	vpdmax4	vpdmax5	vpdmax6	vpdmax7	vpdmax8	vpdmax9	vpdmax10	vpdmax11	vpdmax12
Mean	2.86	3.84	6.78	11.81	16.26	21.03	22.87	21.18	18.68	12.94	6.45	3.27
StD	1.09	1.42	2.11	2.43	3.15	4.11	4.90	4.46	3.95	3.05	1.82	1.08
Min	0.81	1.21	2.16	6.19	8.64	13.19	12.41	12.41	9.04	4.49	2.16	0.89
Q_{25}	2.00	2.76	5.28	9.96	14.07	18.07	19.73	18.04	15.92	11.04	5.13	2.44
Q_{75}	3.52	4.71	8.07	13.48	18.17	23.17	25.11	23.57	20.97	14.43	7.58	3.96
Max	7.18	9.80	15.98	19.94	28.31	37.81	44.68	43.10	36.38	28.28	13.71	7.05
	vpdmin1	vpdmin2	vpdmin3	vpdmin4	vpdmin5	vpdmin6	vpdmin7	vpdmin8	vpdmin9	vpdmin10	vpdmin11	vpdmin12
Mean	0.45	0.52	0.78	1.33	1.59	1.87	1.61	1.16	1.10	0.96	0.72	0.48
StD	0.19	0.21	0.29	0.37	0.49	0.72	0.75	0.65	0.51	0.35	0.23	0.19
Min	0.01	0.01	0.05	0.28	0.06	0.08	0.00	0.00	0.00	0.00	0.07	0.01
Q_{25}	0.30	0.35	0.57	1.09	1.28	1.36	1.10	0.70	0.76	0.74	0.55	0.33
Q_{75}	0.60	0.68	0.97	1.56	1.84	2.25	2.01	1.50	1.35	1.16	0.88	0.63
Max	1.05	1.18	2.04	2.98	4.29	6.39	6.13	5.40	3.82	3.18	1.78	1.32

F Nonlinear relationships between production losses and weather indices

This appendix collects scatterplots of all 72 weather indices against the crop losses, using 1,000 random draws from the sample. The blue curve is fitted by a generalized additive model. The shadow area indicates 95% confidence interval. We can see that most weather indices have intricate nonlinear relationships with crop losses, and this complexity could not be adequately captured by linear models that are used by most existing index insurance design framework. The nonlinearity suggests inadequacy of those index insurance with simple structures, and calls for more sophisticated models.



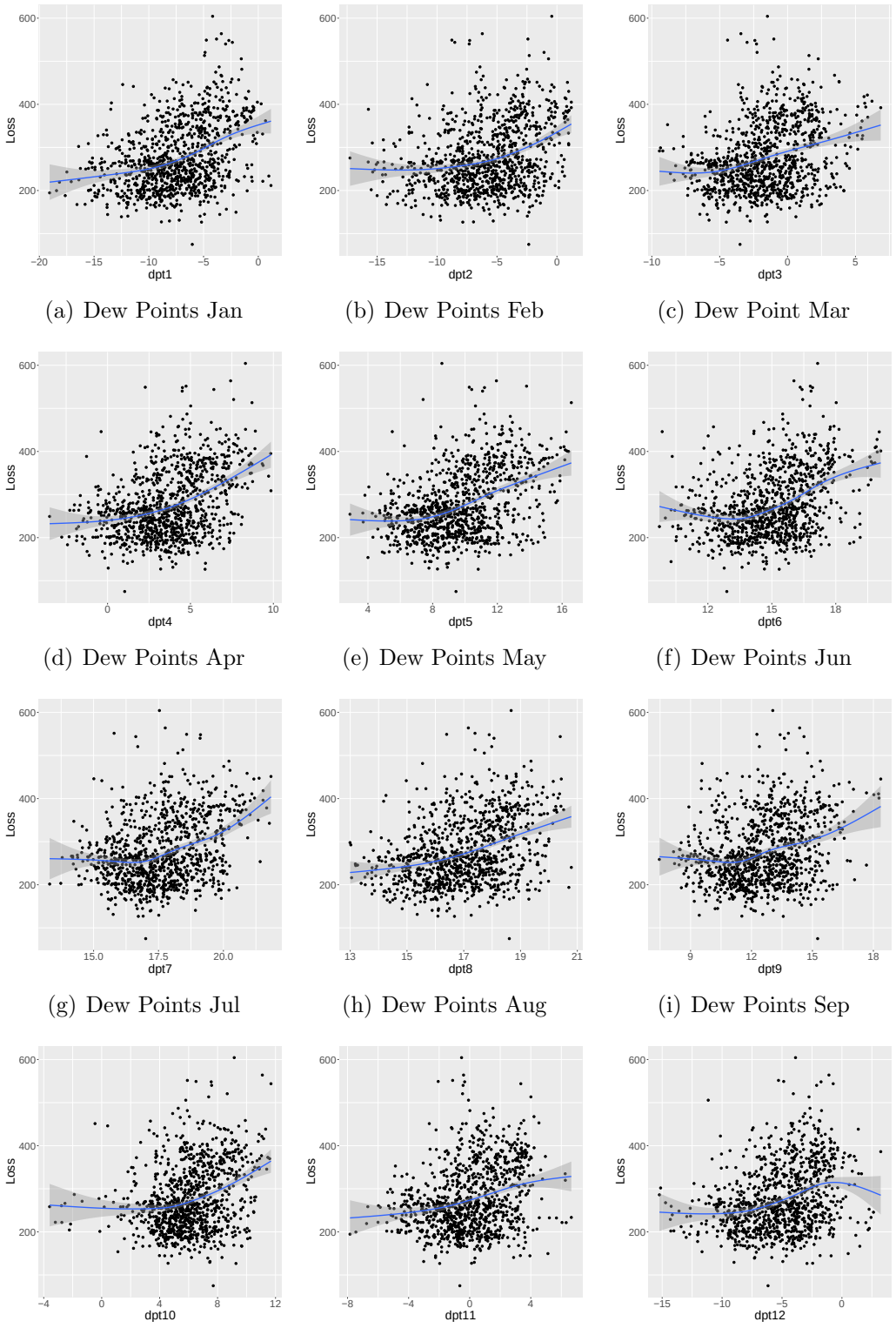
(a) Precipitation Jan (b) Precipitation Feb (c) Precipitation Mar

(d) Precipitation Apr (e) Precipitation May (f) Precipitation Jun

(g) Precipitation Jul (h) Precipitation Aug (i) Precipitation Sep

(j) Precipitation Oct (k) Precipitation Nov (l) Precipitation Dec

Figure F.1: Scatterplots of precipitation (Jan-Dec) with crop losses.



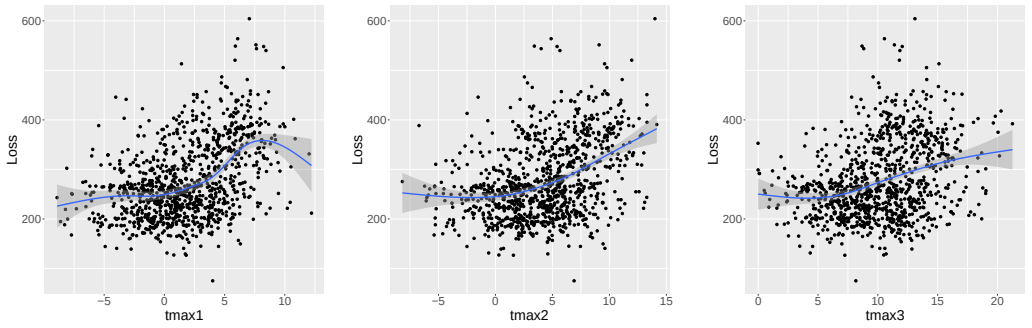
(a) Dew Points Jan (b) Dew Points Feb (c) Dew Point Mar

(d) Dew Points Apr (e) Dew Points May (f) Dew Points Jun

(g) Dew Points Jul (h) Dew Points Aug (i) Dew Points Sep

(j) Dew Points Oct (k) Dew Points Nov (l) Dew Points Dec

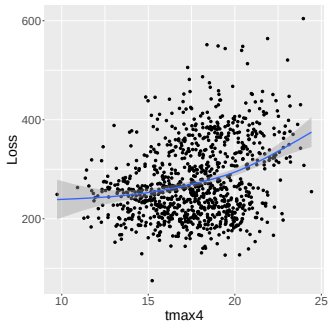
Figure F.2: Scatterplots of dew point temperature (Jan-Dec) with crop losses.



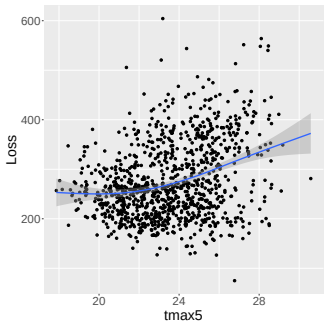
(a) Max Temperature Jan

(b) Max Temperature Feb

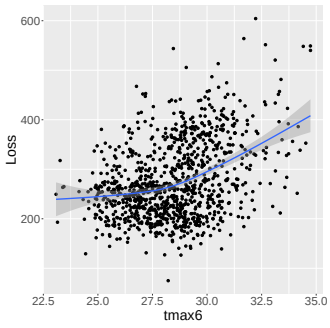
(c) Max Temperature Mar



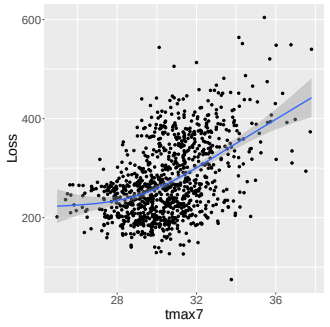
(d) Max Temperature Apr



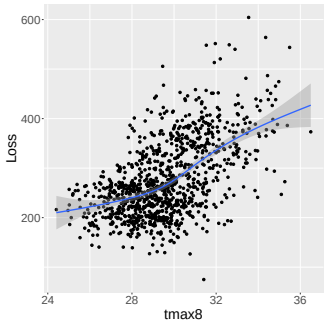
(e) Max Temperature May



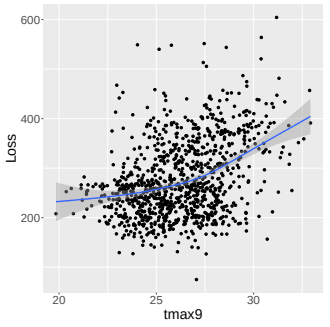
(f) Max Temperature Jun



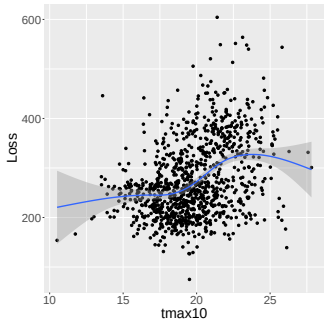
(g) Max Temperature Jul



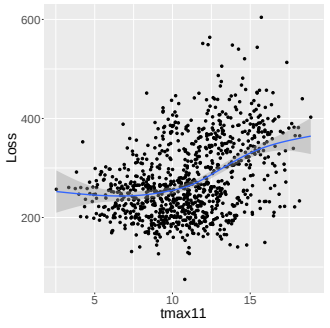
(h) Max Temperature Aug



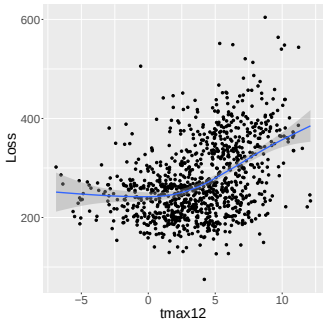
(i) Max Temperature Sep



(j) Max Temperature Oct

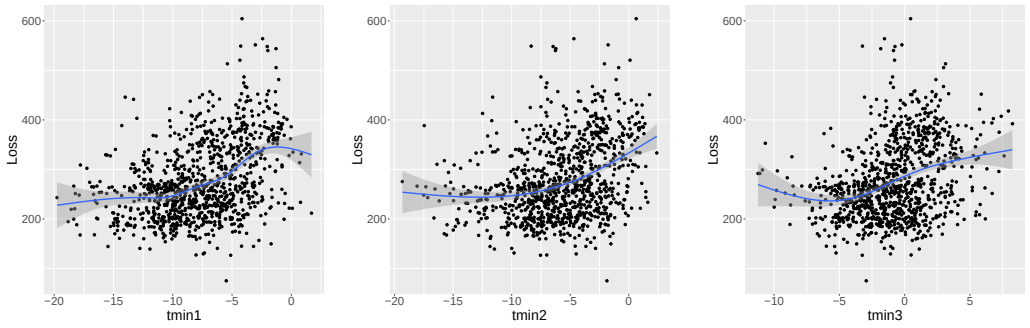


(k) Max Temperature Nov

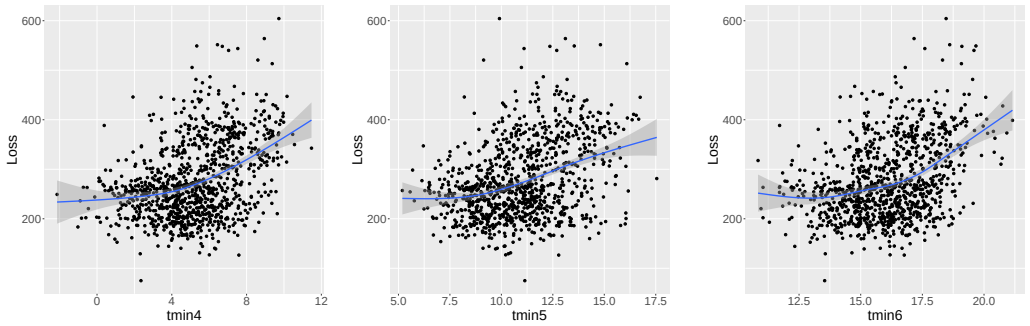


(l) Max Temperature Dec

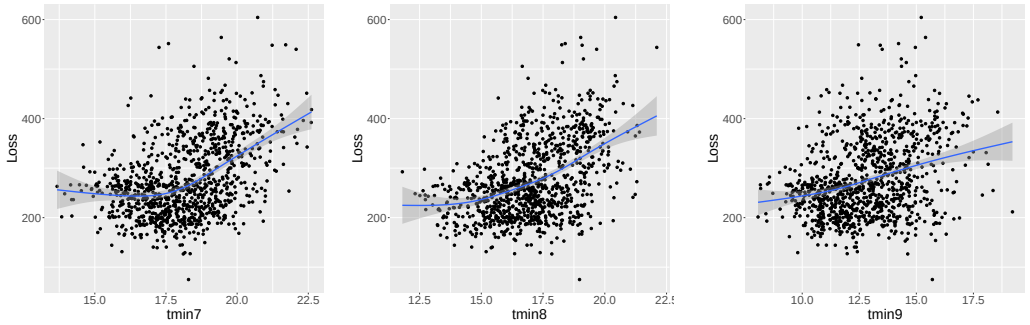
Figure F.3: Scatterplots of maximum temperature (Jan-Dec) with crop losses.



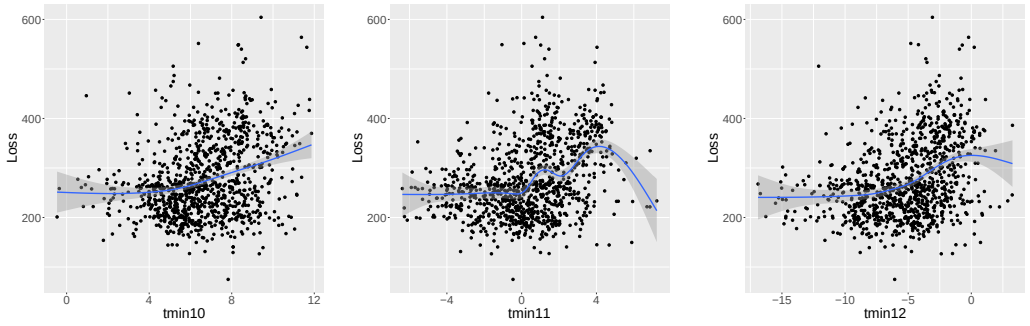
(a) Min Temperature Jan (b) Min Temperature Feb (c) Min Temperature Mar



(d) Min Temperature Apr (e) Min Temperature May (f) Min Temperature Jun

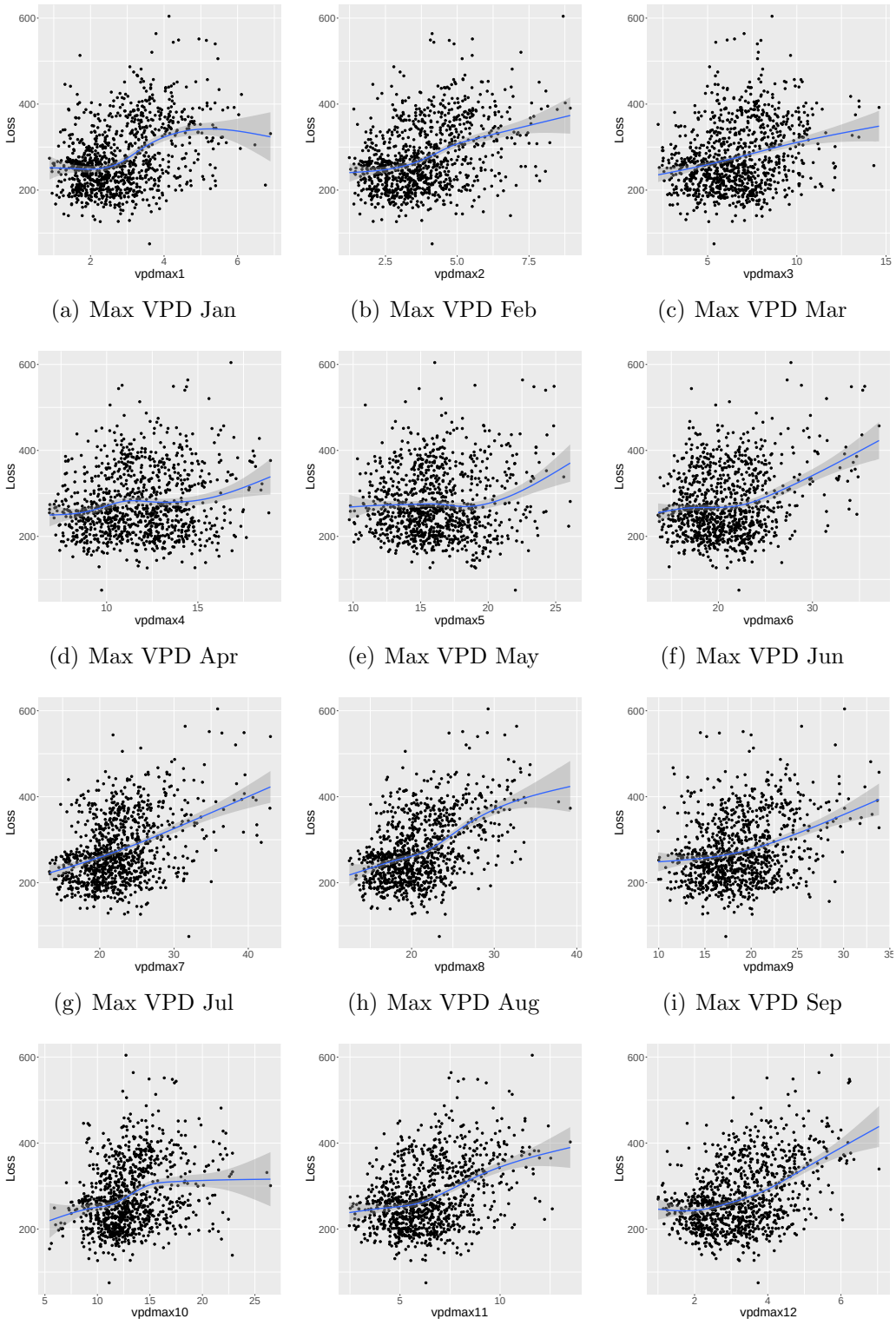


(g) Min Temperature Jul (h) Min Temperature Aug (i) Min Temperature Sep



(j) Min Temperature Oct (k) Min Temperature Nov (l) Min Temperature Dec

Figure F.4: Scatterplots of minimum temperature (Jan-Dec) with crop losses.



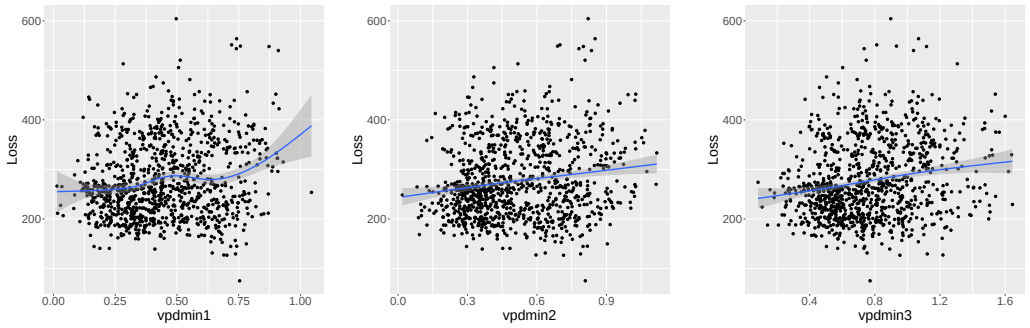
(a) Max VPD Jan (b) Max VPD Feb (c) Max VPD Mar

(d) Max VPD Apr (e) Max VPD May (f) Max VPD Jun

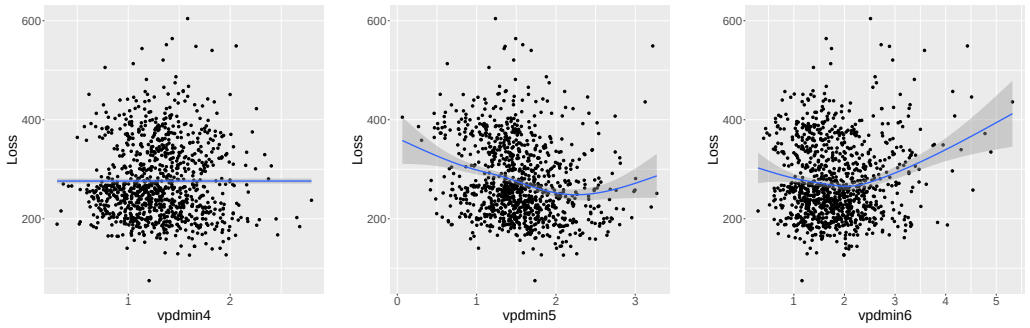
(g) Max VPD Jul (h) Max VPD Aug (i) Max VPD Sep

(j) Max VPD Oct (k) Max VPD Nov (l) Max VPD Dec

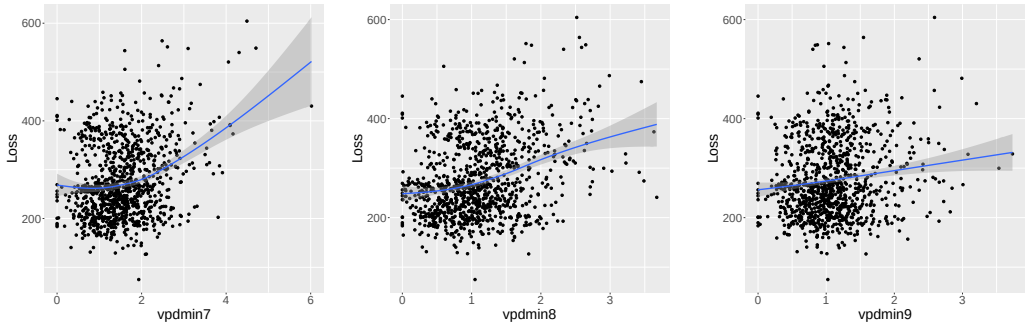
Figure F.5: Scatterplots of maximum vapor pressure deficit (Jan-Dec) with crop losses.



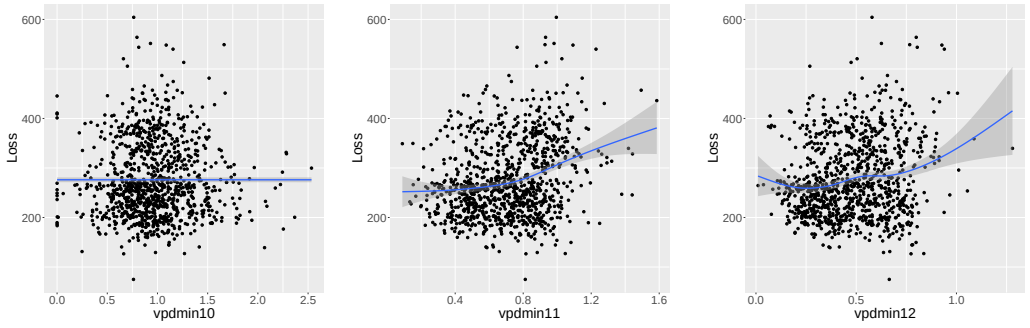
(a) Min VPD Jan (b) Min VPD Feb (c) Min VPD Mar



(d) Min VPD Apr (e) Min VPD May (f) Min VPD Jun



(g) Min VPD Jul (h) Min VPD Aug (i) Min VPD Sep



(j) Min VPD Oct (k) Min VPD Nov (l) Min VPD Dec

Figure F.6: Scatterplots of minimum vapor pressure deficit (Jan-Dec) with crop losses.

G Data homogeneity

Agricultural risk management is often faced with the challenge of data scarcity, since crop yield data are only recorded at annual frequency. Therefore, in order to increase the data sample size, and hence the performance of the trained NN-based index insurance, we assume in this paper that crop yield losses are both time and space homogeneous and expand the sample size to 7,869 county-years. In this appendix, we verify the data homogeneity assumptions. In order to guarantee the time homogeneity, following the literature, we perform a series of statistical analysis to remove trends and heteroscedasticity in the data (see Section 4.1.1 for details). The time homogeneity of our detrended data could be justified by the similarity of data in the three disjoint samples. For example, the utility without insurance in the training, validation, and test samples are -3.99, -3.99, and -4.16, respectively, which are very close. For spatial homogeneity, we perform a simple test by inspecting the homogeneity assumption in the results. In particular, we randomly combine two counties into one location and train the NN-based index insurance again. The results are displayed in Table G.1. We can see the results of the NN-based insurance trained with the randomly combined sample are similar to those trained with the original sample in our main analysis (the performance in the randomly combined sample declines slightly compared to the baseline results because of the reduced sample size). This quantitatively confirms the spatial homogeneity assumption.

Table G.1: **Data homogeneity.** We validate the data homogeneity assumption. In particular, we randomly combine two counties into one location and train the NN-based index insurance again. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" represents the baseline case studied in Section 4.2. The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Sample	Original sample (BL) ($\lambda^* = 1.2413$)		Randomly combined sample ($\lambda^* = 1.2329$)	
Data	Training	Test	Training	Test
Panel A: Utility improvement				
U with insurance	-3.57	-3.57	-3.65	-3.69
U w/o insurance	-3.99	-4.16	-4.07	-4.23
U improvement (%)	10.60%	14.35%	10.51%	12.76%
Panel B: CEW improvement				
CEW with insurance	444.64	444.61	441.85	440.20
CEW w/o insurance	430.63	425.26	427.97	423.14
CEW improvement	14.00	19.36	13.88	17.06
CEW improvement (%)	3.25%	4.55%	3.24%	4.03%
Panel C: Policy characteristics				
Premium	28.44	28.72	29.66	29.08
Coverage	22.91	23.13	24.06	23.59
Insurer Profit	5.53	5.59	5.60	5.49
Panel D: Risk reduction measured by standard deviation				
Std	54.05	47.49	46.07	42.26
Std w/o insurance	81.94	78.92	76.73	74.03
Std reduction	34.04%	39.82%	39.96%	42.92%
Panel E: Risk reduction measured by Value-at-Risk (VaR)				
$\text{VaR}_{5\%}$	382.89	379.64	392.84	390.32
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	320.35	323.69
$\text{VaR}_{5\%}$ improvement	66.61	53.73	72.49	66.63

H Ranking weather indices according to gradient-based sensitivities

This appendix displays ranking of all weather indices based on their gradient-based sensitivities to insurance payoffs from the NN-based index insurance contract and their absolute correlations with production losses, in Figure H.1. The number on top of each bar is the rank difference between using the sensitivity analysis and the absolute correlation. We can see from Figure H.1 that some weather indices are impactful in terms of both absolute correlation and sensitivities (those ranked high with small rank differences, e.g., tmax12, vpdmax8), whereas some weather indices are impactful based on sensitivities but not correlations (those ranked high with larger rank differences, e.g., dpt11, vpdmin5). From the perspective of designing effective index insurance contracts, those weather indices with large absolute value of correlations are not necessarily the most important ones.

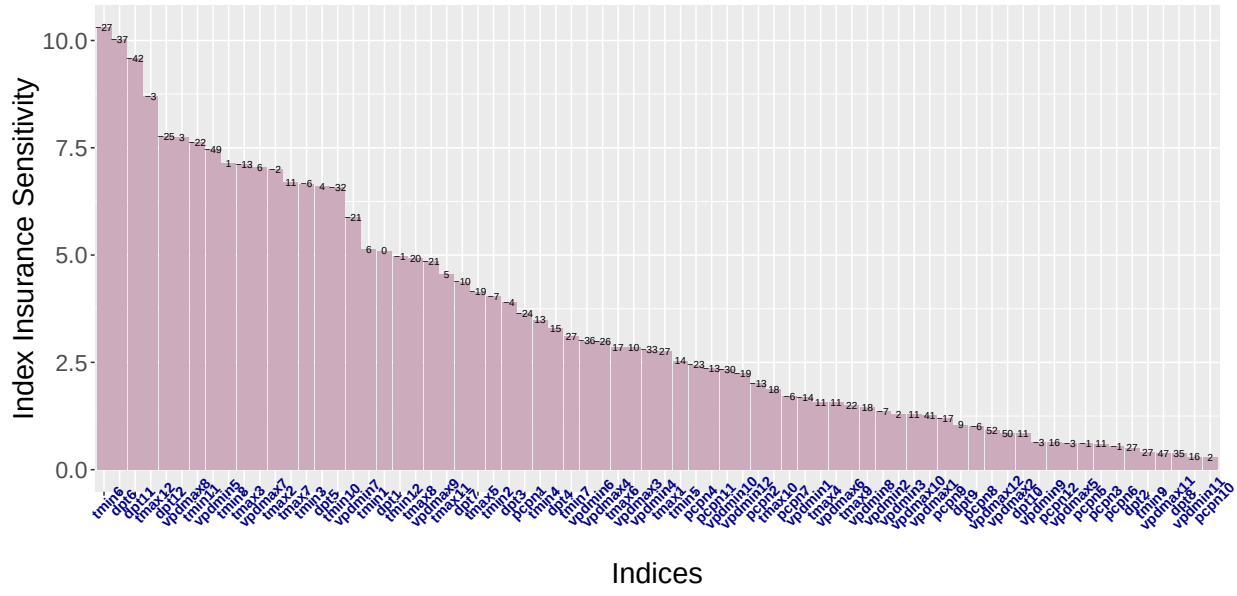
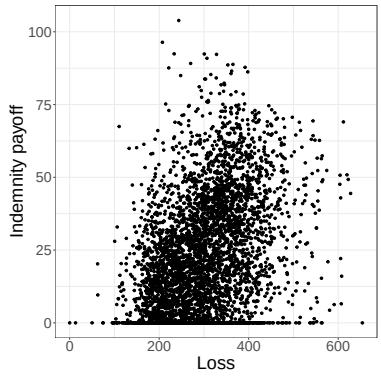


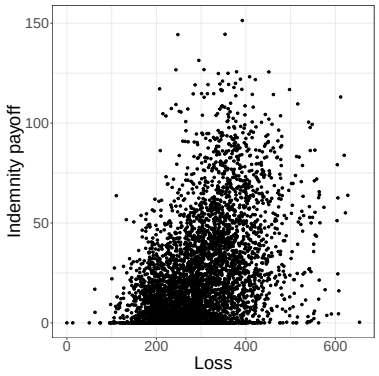
Figure H.1: **Rankings of indices according to index insurance sensitivities.** The number on top of each bar is the rank difference between using the sensitivity analysis criterion and the absolute correlation criterion.

I Basis risk of 7 index insurance contracts

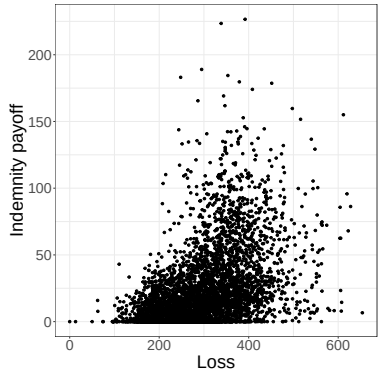
In this subsection, we compare basis risk of seven index insurances considered in Section 4.4. Panel (a) of Figure 1 replicates the large basis risk observed in current practice, which is a single-index piecewise-linear insurance contract (*Linear1*). Figures I.1 and I.2 illustrate how well insurance payoffs match the real losses incurred for the other six index insurance contracts discussed above, using the training sample and test sample, respectively. Across all contracts, except *NN72*, we observe a notably large mismatch between losses and insurance payoffs, especially for the test set. In contrast, *NN72* has a payoff function that is similar to the stop-loss payoff function of a conventional indemnity-based insurance, indicating its dramatic accuracy in mimicking the actual losses by utilizing complex information conveyed in the weather variables. Therefore, the baseline model achieves low basis risk, which is similar to a conventional indemnity-based insurance. These results illustrate the importance of using nonlinear, high-dimensional inputs when designing the index insurance contracts.



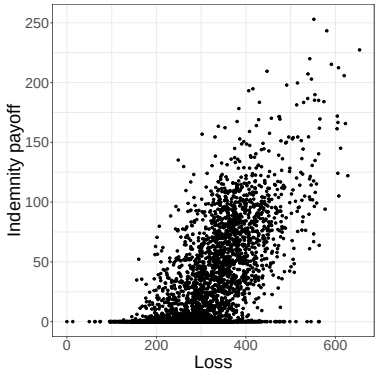
(a) Linear5



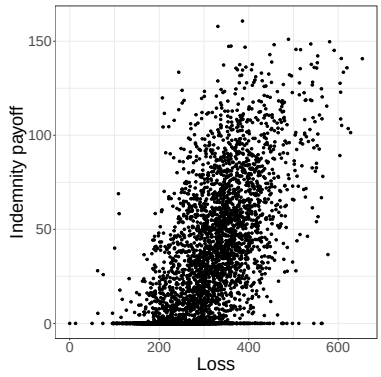
(b) Quadratic5



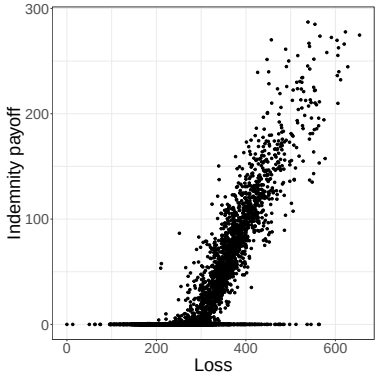
(c) Cubic5



(d) NN5

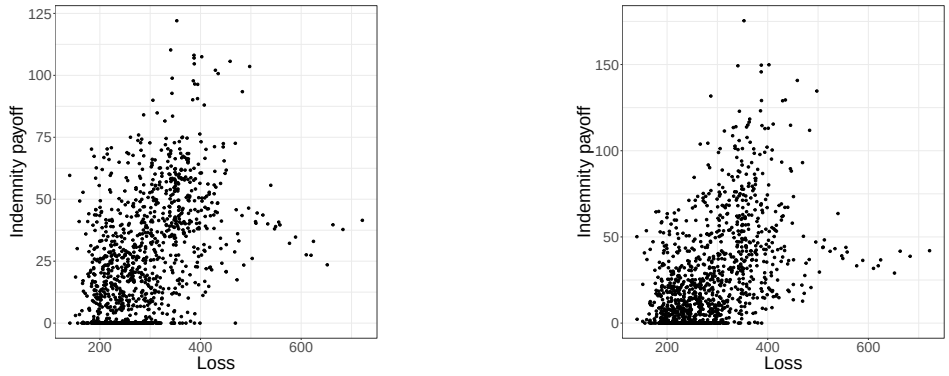


(e) Linear72



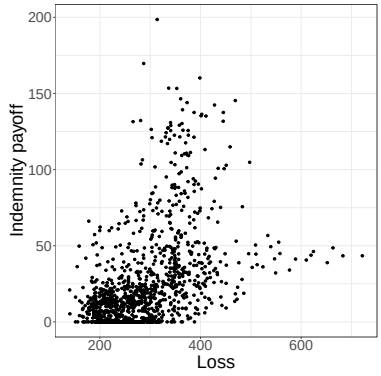
(f) NN72

Figure I.1: **Basis risk of various index insurance contracts, using the training sample.** These panels plot the insurance payoffs against actual loss, using the training sample. Six insurance contracts are presented, including (a) a linear insurance contract with five weather indices (Linear5); (b) a quadratic insurance contract with five weather indices (Quadratic5); (c) a cubic insurance contract with five weather indices (Cubic5); (d) an NN-based contract with five weather indices (NN5); (e) a linear insurance contract with 72 weather indices (Linear72); and (f) the baseline model (NN72, an NN-based contract with 72 weather indices).

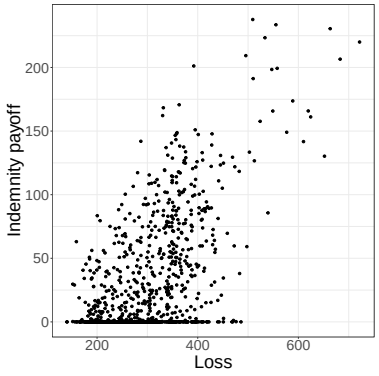


(a) Linear5

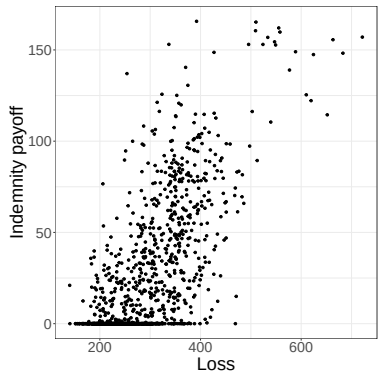
(b) Quadratic5



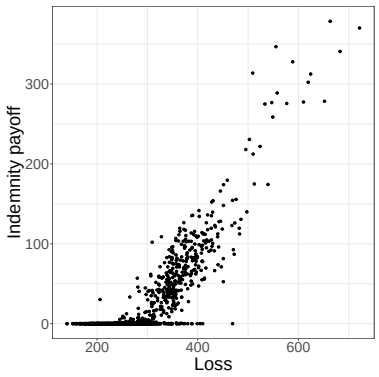
(c) Cubic5



(d) NN5



(e) Linear72



(f) NN72

Figure I.2: **Basis risk of various index insurance contracts, using the test sample.** These panels plot the insurance payoffs against actual loss, using the test sample. Six insurance contracts are presented, including (a) a linear insurance contract with five weather indices (Linear5); (b) a quadratic insurance contract with five weather indices (Quadratic5); (c) a cubic insurance contract with five weather indices (Cubic5); (d) an NN-based contract with five weather indices (NN5); (e) a linear insurance contract with 72 weather indices (Linear72); and (f) the baseline model (NN72, an NN-based contract with 72 weather indices).

J The impacts of dimensionality

In this subsection, we investigate the impact of dimensionality on the NN-based index insurance performance. We consider models with the most important 1, 18, 36, 54, and 72 weather indices. The index importance is ranked based on the gradient-based sensitivity analysis discussed in Section 4.3. We see that using only one index improves the utility by 0.47% in the test set. Adding more weather indices significantly improves the model performances. For example, the model with 36 weather indices improves the utility by 13.40% in the test set. This analysis demonstrates the importance of including higher dimensional inputs in the NN-based index insurance contract.

Table J.1: **Comparing models with various number of weather indices.** We evaluate the performance of the NN-based index insurance with different number of weather indices in the test set, using 1, 18, 36, 54, and 72 weather indices. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" represents the baseline case studied in Section 4.2.

	72 indices (BL)	54 indices	36 indices	18 indices	One index
Panel A: Utility improvement					
U with insurance	-3.57	-3.60	-3.61	-3.67	-4.14
U w/o insurance	-4.16	-4.16	-4.16	-4.16	-4.16
U improvement (%)	14.35%	13.64%	13.40%	11.90%	0.47%
Panel B: CEW improvement					
CEW with insurance	444.61	443.58	443.24	441.10	425.84
CEW w/o insurance	425.26	425.26	425.26	425.26	425.26
CEW improvement	19.36	18.33	17.99	15.84	0.58
CEW improvement (%)	4.55%	4.31%	4.23%	3.73%	0.14%
Panel C: Policy characteristics					
Premium	28.72	28.56	27.12	21.61	27.24
Coverage	23.13	23.01	21.85	17.41	25.91
Insurer Profit	5.59	5.55	5.27	4.20	1.33
Panel D: Risk reduction measured by standard deviation					
Std	47.49	49.46	50.94	56.63	72.92
Std w/o insurance	78.92	78.92	78.92	78.92	78.92
Std reduction	39.82%	37.33%	35.45%	28.24%	7.60%
Panel E: Risk reduction measured by Value-at-Risk (VaR)					
$\text{VaR}_{5\%}$	379.64	371.54	370.77	357.98	335.30
$\text{VaR}_{5\%}$ w/o insurance	325.91	325.91	325.91	325.91	325.91
$\text{VaR}_{5\%}$ improvement	53.73	45.64	44.86	32.08	9.39

K Weather predictability

In the past four decades, numerical weather prediction technology has been improving a lot. That is, adding one day of predictive power per decade (Bauer et al., 2015). However, long-term (e.g., several months or one-year ahead) weather is still unpredictable (Alley et al., 2019; Voosen, 2019). For example, Figure K.1 plots the forecast skill at three-, five-, seven-, and ten-day ranges. The best forecast at the European Centre for Medium-Range Weather Forecasts (ECMWF) runs out to around 10 days. In fact, research shows that there indeed exists a predictability limit for weather forecast, which is 4-5 days in general and 10 days for midlatitude weather.

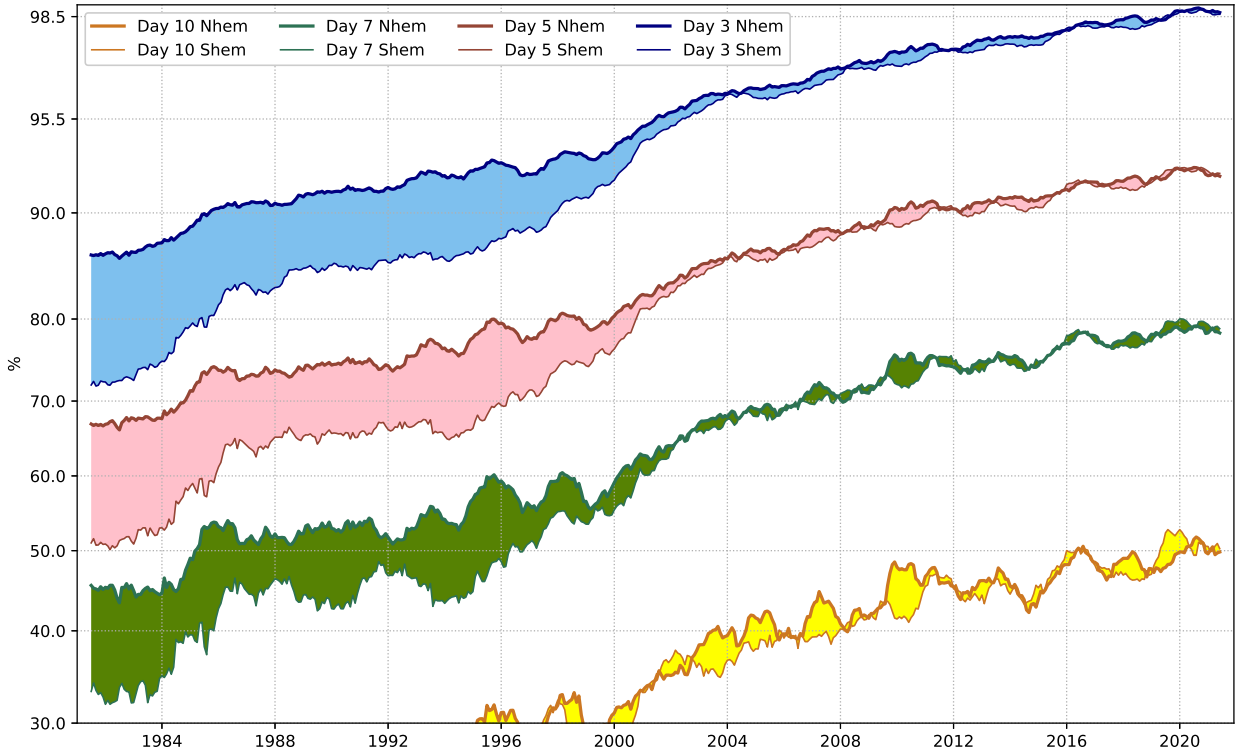


Figure K.1: **The evolution of weather forecast quality.** This figure plots the forecast skill at three-, five-, seven-, and ten-day ranges. The forecast skill is measured by anomaly correlation coefficient (ACC) of the height of 500-hPa level between the forecasts and observations. The two curves are computed over the extra-tropical northern and southern hemispheres. In practice, a value higher than 60% is treated as a skillful weather forecast. This plot is adapted from the ECMWF official website (`https://www.ecmwf.int/en/forecasts/charts/catalogue/plwww_m_hr_ccaf_adrian_ts`).

L Robustness checks

We further perform several robustness checks in this section. First, we examine the impacts of insurers' characteristics. We consider various insurers' supply curves and also exogenously given risk loading. Second, we examine the impacts of farmers' characteristics, such as different coverage levels, risk aversion, and alternative utility functions (e.g., log utility and power utility) on the NN-based index insurance contract.

L.1 Different insurers' supply curves

In our previous analysis, the equilibrium loading parameter, λ^*, was determined via a reduced-form approach, where the supply curve is estimated using market data from the USDA SOB Reports, which might have simultaneity issue. To address these concerns, in this subsection, we further investigate the robustness of insurers' supply curve. We use the upper and lower bounds of [10%, 90%] and [25%, 75%] confidence intervals of the supply curve estimates to determine the equilibrium loading parameter. Figure L.1 displays our estimated supply curve with its confidence intervals.

Table L.1 summarizes the results. When insurers' supply curve shifts to the upper bounds of its confidence intervals, equilibrium loading parameter, λ^*, decreases, and the equilibrium insurance demand increases. As a consequence, farmers buy more index insurance, and achieve more utility improvements and higher CEW. While insurance demand increases, we observe that insurers do not gain higher profits as the insurance is priced lower. When the insurance supply curve moves toward the lower bounds of its confidence intervals, λ^* increases and insurance demand decreases. Correspondingly, farmers' utilities and CEW improvements are reduced. Overall, under various supply curves, the NN-based index insurance contract provides robust results for utility and CEW improvements, risk reduction, and insurers' profits. Therefore, potential simultaneity issue barely affects the results.

L.2 Exogenous risk loading and market demand

Previously, we mainly analyze the equilibrium when insurance premium is endogenously determined. However, it is possible that there are some exogenous sources affecting the insurance premium. For example, supply frictions such as administration costs, litigation risks and regulatory frictions that insurers might face, or imperfect competition among insurers. This can be captured by a partial equilibrium case where insurers can choose risk loading, λ. To this end, we consider exogenous values of $\lambda = 1.33, 1.37$, and 1.415, which corresponds to the case when insurance demand reduces by $20\%, 30\%$, and 40% relative to the endogenous case, respectively. The results are summarized in Table L.2. We see that as the insurance become more expensive, both the farmer's incentive to purchase insurance and utility improvement decrease. However, it is important to note that even with the largest demand reduction of 40%, the policyholder gains a utility improvement of 11.31% and a CEW improvement of \$15 in the test sample, and basis risk is significantly reduced, as measured by either standard deviation or VaR. Finally, the insurer is observed to have a trade-off when determining its risk loading. While a larger λ leads to a higher profit margin, it negatively affects its market demand and thus the total profit.

L.3 Farmers with different coverage demands

In the baseline case, we focus on farmers who are solely interested in maximizing their utility, regardless of the coverage they purchase and premiums they pay. In practice, however, farmers often have a predetermined level of coverage in mind, because of either a better understanding of their financial position and insurance demand, or a relatively tight budget constraint. As a result, these farmers may be interested in more customized index insurance contracts. The NN-based index insurance design proposed in this paper is convenient to create customized contracts to meet their demands.

For illustration purposes, we consider a set of index insurance plans with a coverage of \$10, \$20, \$30, and \$40. Table L.3 summarizes the results of these four contracts. For comparison

purposes, we also list the baseline model which has the optimal coverage of \$23.13. We see that the amount of utility improvement first increases with coverage, peaking in the baseline case, and then decreasing with the coverage level. Overall, the NN-based insurance contract provides reasonable utility improvement for various coverage levels.

L.4 Farmers with different levels of risk aversion

Farmers' risk aversion varies with their age, education, farming experience, wealth, etc. One might wonder how different risk appetites lead to different demands for insurance. In this subsection, we consider policyholders with various levels of risk aversion. In addition to the baseline case in which $\alpha = 0.008$ (corresponding to a relative risk aversion of 3.1), we consider farmers with relative risk aversions of $2, 4$, and 5, which correspond to absolute risk aversion coefficients of $\alpha = 0.0051, 0.0103$, and 0.0129, respectively. Table L.4 summarizes the results. Table L.4 shows that the optimal index insurance design achieves greater utility and CEW improvements for farmers with higher risk aversion. This is because more risk-averse policyholders are more concerned about volatilities in their wealth. As such, insurers can charge these farmers higher premiums (i.e., imposing a higher loading parameter, λ^*).

Next, we consider farmers with time-varying risk aversion which depends on losses in the previous year. For example, farmers might become more risk averse after large losses, especially for less educated farmers without long-term learning skills (Cai et al., 2020). Such time-varying risk aversion might capture time inconsistency as well. Specifically, we first compute the 75^{th} and 25^{th} percentiles of yield loss. Suppose the farmer's average relative risk aversion is $RRA = 3.1$ (absolute risk aversion is $\alpha = 0.008$). If the farmer experiences a loss larger than the 75^{th} percentile in year $t-1$, her risk aversion in year t becomes $3.1\times(1+x)$; on the contrary, if the farmer experiences a loss lower than the 25^{th} percentile in year $t-1$, her risk aversion in year t is $3.1\times(1-x)$. That is, the farmer's risk aversion is $3.1\times(1-x), 3.1$, or $3.1 \times (1 + x)$, depending on the previous loss experience. We consider different levels of risk aversion variations, i.e., $x = 0.1, 0.2$, and 0.3. The results are summarized in Table L.5.

Generally, we see that the NN-based index insurance consistently improves farmers' utility and CEW, and reduces basis risk across the specifications. Nevertheless, time-varying risk aversion impedes the performance of the designed index insurance, with larger variations of risk aversion hindering the insurance performance more.

L.5 Alternative utility functions

In this subsection, we evaluate the performance of the proposed NN-based index insurance using constant relative risk aversion (CRRA) utility functions. We consider the power utility with various levels of risk aversion, that is, relative risk aversion (RRA) of 2, 3, 4, and 5, and log utility (RRA = 1). Again, we use the 3-hidden-layer (64-64-16 neurons) structure, as in the baseline model. Table L.6 summarizes the results.[26] The performance is similar to the baseline case with negative exponential utility. Using log utility, we find that the farmer's utility and CEW improvements are marginal because the risk aversion of her log utility is low (RRA = 1). Evaluating the results of power utility, we see that as policyholders become more risk averse, they purchase more coverage, and insurers also make higher profits. Index insurance performance also significantly increases with risk aversion. For example, the CEW improvement for RRA = 5 is about five times larger than that for RRA = 2. Comparing the results with the negative exponential utility case in Table L.4, we observe that the insurance has higher CEW improvements, given the same RRA. This is because power utility functions penalize extremely low wealth cases more severely.

[26]Logarithm utility and power utility functions are not defined for negative wealth. Therefore, to avoid the negative wealth cases, we winsorize the loss data at 99% percentile.

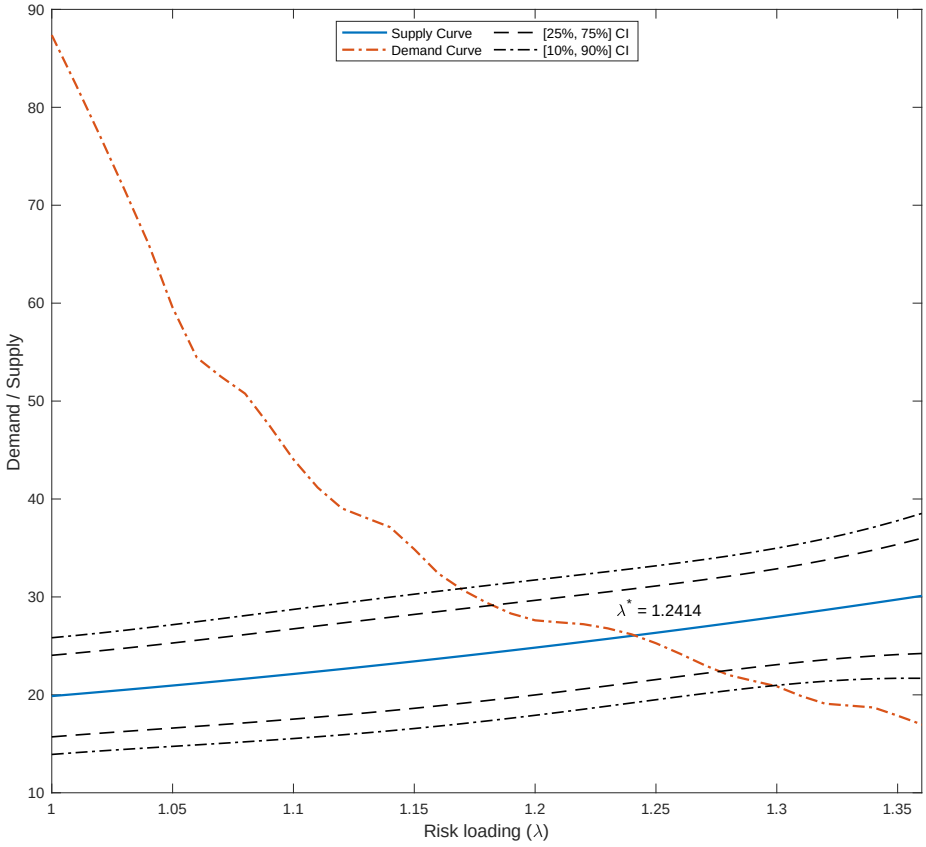


Figure L.1: **Confidence intervals of the insurance supply curve.** This figure displays the upper and lower bounds of the [10%, 90%] and [25%, 75%] confidence intervals for the estimated supply curve of the index insurance. The insurance supply curve is fitted from the USDA SOB Reports data with a power function using the nonlinear least squares method. The demand curve is for the NN-based optimal index insurance with a 3-hidden-layer (64-64-16 neurons) structure, and is fitted with a piecewise cubic hermite interpolating polynomial.

Table L.1: **Impacts of insurer's supply curves.** We test the robustness of our results using the upper and lower bounds of [10%, 90%] and [25%, 75%] confidence intervals (CI) of the supply curve estimates. Panel A summarizes utilities with and without (w/o) index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance and the CEW improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurer. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Supply curve	[25%, 75%] CI				[10%, 90%] CI			
	Lower bound		Upper bound		Lower bound		Upper bound	
	($\lambda^* = 1.2814$)		($\lambda^* = 1.1871$)		($\lambda^* = 1.3142$)		($\lambda^* = 1.1744$)	
Data	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement								
U with insurance	-3.57	-3.65	-3.53	-3.53	-3.59	-3.67	-3.52	-3.52
U w/o insurance	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16
U improvement (%)	10.43%	12.34%	11.53%	15.31%	9.95%	11.81%	11.77%	15.54%
Panel B: CEW improvement								
CEW with insurance	444.41	441.72	445.95	446.03	443.73	440.96	446.28	446.37
CEW w/o insurance	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26
CEW improvement	13.77	16.46	15.32	20.77	13.10	15.70	15.65	21.11
CEW improvement (%)	3.20%	3.87%	3.56%	4.89%	3.04%	3.69%	3.63%	4.96%
Panel C: Policy characteristics								
Premium	26.28	26.84	29.38	30.07	26.57	25.92	31.38	31.73
Coverage	20.51	20.95	24.75	25.33	20.21	19.72	26.72	27.02
Insurer Profit	5.77	5.90	4.63	4.74	6.35	6.20	4.66	4.71
Panel D: Risk reduction measured by standard deviation								
Std	54.82	53.27	52.86	45.89	55.06	54.09	51.78	44.94
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	33.10%	32.49%	35.49%	41.85%	32.81%	31.46%	36.81%	43.06%
Panel E: Risk reduction measured by Value-at-Risk (VaR)								
$\text{VaR}_{5\%}$	384.10	362.39	384.30	383.05	383.67	358.27	385.54	384.74
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	67.82	36.49	68.03	57.14	67.39	32.36	69.27	58.83

Table L.2: **Exogenously specified risk loading.** This table compares insurance contract performances when risk loading is exogenously specified. We consider some exogenously given risk loadings: $\lambda^* = 1.33, 1.37$ and 1.415, which corresponds to a reduction of demand by 20%, 30%, and 40%, relative to the baseline model. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" represents the baseline case studied in Section 4.2.

Coverage reduction	0% (BL)		20%		30%		40%	
Risk loading	$\lambda = 1.2414$ (BL)		$\lambda = 1.33$		$\lambda = 1.37$		$\lambda = 1.415$	
Data	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement								
U with insurance	-3.57	-3.57	-3.63	-3.64	-3.65	-3.68	-3.68	-3.69
U w/o insurance	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16
U improvement (%)	10.60%	14.35%	9.03%	12.60%	8.39%	11.60%	7.84%	11.31%
Panel B: CEW improvement								
CEW with insurance	444.64	444.61	442.46	442.10	441.58	440.67	440.84	440.26
CEW w/o insurance	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26
CEW improvement	14.00	19.36	11.82	16.84	10.95	15.41	10.21	15.00
CEW improvement (%)	3.25%	4.55%	2.75%	3.96%	2.54%	3.62%	2.37%	3.53%
Panel C: Policy characteristics								
Premium	28.44	28.72	24.43	24.37	22.06	20.02	19.33	17.18
Coverage	22.91	23.13	18.37	18.32	16.10	14.61	13.66	12.14
Insurer Profit	5.53	5.59	6.06	6.05	5.96	5.41	5.67	5.04
Panel D: Risk reduction measured by standard deviation								
Std	54.05	47.49	57.50	52.16	59.64	56.42	61.88	58.30
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	34.04%	39.82%	29.83%	33.90%	27.22%	28.50%	24.48%	26.13%
Panel E: Risk reduction measured by Value-at-Risk (VaR)								
$\text{VaR}_{5\%}$	382.89	379.64	379.56	371.51	373.37	362.77	369.05	362.05
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	66.61	53.73	63.28	45.61	57.09	36.86	52.78	36.14

Table L.3: **Impacts of coverage level.** We consider an NN-based index insurance with various coverage levels. Panel A summarizes utilities with and without (w/o) index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance policies and CEW improvements in dollars and as a percentage. Panel C summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel D summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" indicates the baseline case studied in Section 4.2.

	Coverage: \$10		Coverage: \$20		Coverage: \$23.13(BL)		Coverage: \$30		Coverage: \$40	
Data	Training	Test	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement										
U with insurance	-3.63	-3.66	-3.56	-3.58	-3.57	-3.57	-3.58	-3.58	-3.62	-3.64
U w/o insurance	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16
U improvement (%)	8.98%	12.02%	10.60%	14.11%	10.60%	14.35%	10.34%	13.95%	9.21%	12.54%
Panel B: CEW improvement										
CEW with insurance	442.40	441.26	444.65	444.28	444.64	444.61	444.28	444.04	442.71	442.00
CEW w/o insurance	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26
CEW improvement	11.76	16.00	14.01	19.02	14.00	19.36	13.65	18.78	12.07	16.75
CEW improvement (%)	2.73%	3.76%	3.25%	4.47%	3.25%	4.55%	3.17%	4.42%	2.80%	3.94%
Panel C: Risk reduction measured by standard deviation										
Std	65.61	61.72	56.03	50.21	54.05	47.49	50.71	43.94	47.97	42.08
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	19.94%	21.79%	31.62%	36.37%	34.04%	39.82%	38.11%	44.32%	41.46%	46.68%
Panel D: Risk reduction measured by Value-at-Risk (VaR)										
$\text{VaR}_{5\%}$	359.65	353.98	380.75	374.99	382.89	379.64	383.57	382.84	380.75	379.06
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	43.37	28.07	64.47	49.09	66.61	53.73	67.29	56.93	64.47	53.16

Table L.4: **Impacts of risk aversion.** We consider an NN-based index insurance for farmers with various levels of risk aversion, i.e., relative risk aversion (RRA) of 2, 3.1, 4, and 5. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" represents the baseline case studied in Section 4.2. The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Risk aversion	$\alpha = 0.0051$ (RRA = 2) ($\lambda^* = 1.1753$)		$\alpha = 0.008$ (BL, RRA = 3.1) ($\lambda^* = 1.2414$)		$\alpha = 0.0103$ (RRA = 4) ($\lambda^* = 1.2744$)		$\alpha = 0.0129$ (RRA = 5) ($\lambda^* = 1.2876$)	
Data	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement								
U with insurance	-18.47	-18.64	-3.57	-3.57	-1.04	-1.04	-0.28	-0.28
U w/o insurance	-19.23	-19.53	-3.99	-4.16	-1.29	-1.42	-0.41	-0.51
U improvement (%)	3.96%	4.57%	10.60%	14.35%	19.13%	26.94%	31.37%	45.87%
Panel B: CEW improvement								
CEW with insurance	450.60	448.79	444.64	444.61	440.20	440.63	435.24	436.75
CEW w/o insurance	442.84	439.80	430.63	425.26	419.59	410.16	406.06	389.18
CEW improvement	7.76	8.99	14.00	19.36	20.61	30.47	29.18	47.58
CEW improvement (%)	1.75%	2.05%	3.25%	4.55%	4.91%	7.43%	7.19%	12.22%
Panel C: Policy characteristics								
Premium	25.34	24.14	28.44	28.72	29.56	29.42	30.65	31.31
Coverage	21.56	20.54	22.91	23.13	23.20	23.09	23.80	24.32
Insurer Profit	3.78	3.60	5.53	5.59	6.36	6.33	6.85	6.99
Panel D: Risk reduction measured by standard deviation								
Std	54.63	51.10	54.05	47.49	53.69	48.31	53.33	48.12
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	33.33%	35.25%	34.04%	39.82%	34.48%	38.78%	34.92%	39.03%
Panel E: Risk reduction measured by Value-at-Risk (VaR)								
$\text{VaR}_{5\%}$	385.46	372.67	382.89	379.64	382.63	374.71	382.88	371.75
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	69.19	46.76	66.61	53.73	66.35	48.80	66.60	45.84

Table L.5: **Impact of time-varying risk aversion.** We evaluate the index insurance performance with time-varying risk aversion. The farmer's average relative risk aversion is $RRA = 3.1$. If the farmer experiences a loss larger than the 75^{th} percentile in year $t-1$, her risk aversion in year t becomes $3.1 \times (1+x)$; on the contrary, if the farmer experiences a loss lower than the 25^{th} percentile in year $t-1$, her risk aversion in year t is $3.1 \times (1-x)$. Columns 2-7 display results for different risk aversion variations ($x = 0.1, 0.2,$ and 0.3). The last two columns correspond to a constant risk aversion of 3.1, which is our baseline model. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR).

Risk aversion	$x = 0.1$ $RRA = 2.8, 3.1, 3.4$		$x = 0.2$ $RRA = 2.5, 3.1, 3.7$		$x = 0.3$ $RRA = 2.2, 3.1, 4$		Baseline model $RRA = 3.1$	
Data	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement								
U with insurance	-3.75	-3.79	-4.41	-4.51	-5.74	-5.94	-3.57	-3.57
U w/o insurance	-4.18	-4.33	-4.86	-4.99	-6.22	-6.33	-3.99	-4.16
U improvement (%)	10.22%	12.57%	9.23%	9.62%	7.81%	6.17%	10.60%	14.35%
Panel B: CEW improvement								
CEW with insurance	444.82	443.35	443.97	440.88	442.35	438.13	444.64	444.61
CEW w/o insurance	431.21	425.35	431.22	424.60	431.17	423.60	430.63	425.26
CEW improvement	13.61	18.00	12.75	16.27	11.18	14.54	14.00	19.36
CEW improvement (%)	3.16%	4.23%	2.96%	3.83%	2.59%	3.43%	3.25%	4.55%
Panel C: Policy characteristics								
Premium	28.24	28.56	26.05	25.83	25.35	24.60	28.44	28.72
Coverage	22.75	23.01	20.99	20.81	20.42	19.82	22.91	23.13
Insurer Profit	5.49	5.56	5.07	5.02	4.93	4.78	5.53	5.59
Panel D: Risk reduction measured by standard deviation								
Std	54.65	51.17	57.18	56.55	60.02	61.74	54.05	47.49
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	33.30%	35.16%	30.21%	28.34%	26.76%	21.77%	34.04%	39.82%
Panel E: Risk reduction measured by Value-at-Risk (VaR)								
$\text{VaR}_{5\%}$	379.98	367.23	370.58	357.64	359.41	348.69	382.89	379.64
$\text{VaR}_{5\%}$ w/o insurance	316.59	326.59	316.59	326.59	316.59	326.59	316.28	325.91
$\text{VaR}_{5\%}$ improvement	63.39	40.64	53.99	31.05	42.82	22.10	66.61	53.73

Table L.6: **Alternative utility functions.** We consider power utility with various levels of risk aversion, i.e., relative risk aversion (RRA) of 2, 3, 4, and 5 and log utility (RRA = 1). The NN uses a 3-hidden-layer (64-64-16 neurons) structure. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance policies and CEW improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Utility function	Log Utility		Power Utility							
Risk aversion	RRA = 1 ($\lambda^* = 1.0894$)		RRA = 2 ($\lambda^* = 1.1546$)		RRA = 3 ($\lambda^* = 1.2048$)		RRA = 4 ($\lambda^* = 1.2567$)		RRA = 5 ($\lambda^* = 1.2849$)	
Data	Training	Test	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement										
U with insurance	6.12	6.12	-2.21×10^{-03}	-2.22×10^{-03}	-2.51×10^{-06}	-2.51×10^{-06}	-3.89×10^{-09}	-3.84×10^{-09}	-7.11×10^{-12}	-6.72×10^{-12}
U w/o insurance	6.12	6.11	-2.26×10^{-03}	-2.28×10^{-03}	-2.7×10^{-06}	-2.84×10^{-06}	-4.74×10^{-09}	-5.35×10^{-09}	-10.6×10^{-12}	-13.12×10^{-12}
U improvement (%)	0.11%	0.15%	1.88%	2.70%	7.30%	11.84%	17.88%	28.25%	32.88%	48.76%
Panel B: CEW improvement										
CEW with insurance	456.89	454.70	451.87	450.47	446.76	446.59	440.89	442.75	432.97	439.11
CEW w/o insurance	453.74	450.45	443.39	438.30	430.13	419.31	412.88	396.37	391.90	371.51
CEW improvement	3.15	4.25	8.47	12.17	16.62	27.28	28.02	46.38	41.07	67.60
CEW improvement (%)	0.70%	0.94%	1.91%	2.78%	3.86%	6.51%	6.79%	11.70%	10.48%	18.20%
Panel C: Policy characteristics										
Premium	20.72	21.56	23.79	24.68	24.99	25.68	26.76	28.91	30.90	32.87
Coverage	19.02	19.79	20.60	21.37	20.74	21.32	21.29	23.00	24.05	25.58
Insurer Profit	1.70	1.77	3.19	3.31	4.25	4.37	5.47	5.91	6.85	7.29
Panel D: Risk reduction measured by standard deviation										
Std	56.79	50.38	55.59	49.18	55.53	49.03	55.17	47.52	54.37	46.03
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	30.69%	36.17%	32.16%	37.68%	32.23%	37.88%	32.67%	39.78%	33.64%	41.67%
Panel E: Risk reduction measured by Value-at-Risk (VaR)										
$\text{VaR}_{5\%}$	381.59	377.18	382.78	377.83	381.08	377.95	380.19	380.29	376.03	378.72
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	65.31	51.27	66.50	51.92	64.80	52.04	63.92	54.38	59.75	52.81

M Protecting corn price risk

Previously we focus on discussing index insurance for production losses, i.e., yield insurance. However, as corn prices fluctuate, one might consider simultaneously providing corn price protection to farmers. In this section, we apply the same NN-based framework and design an index insurance contract to protect both the production and price risks, that is, the revenue protection.

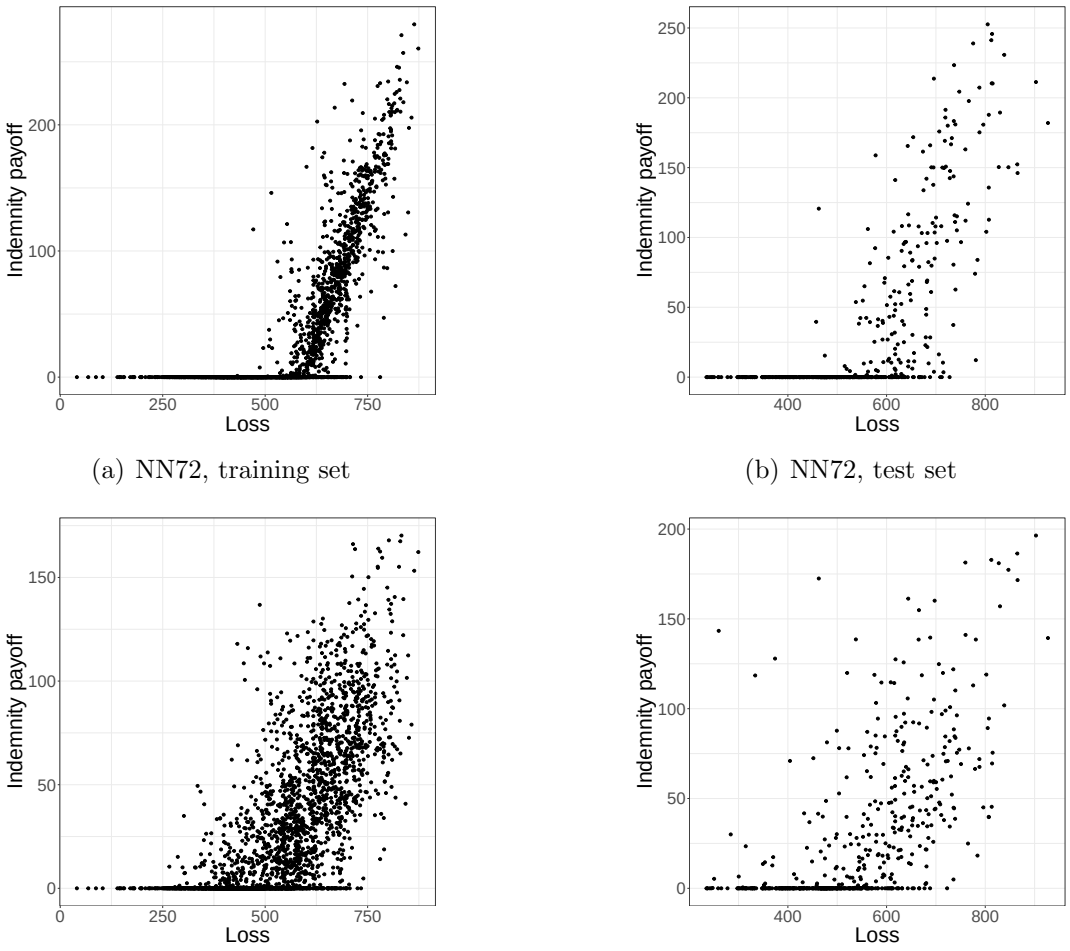
We use the average price of the Chicago Mercantile Exchange (CME) Group December futures contracts during the month of February as the expected corn price.[27] The futures price contains the market's expectation of the corn commodity demand and supply within the same calendar year. In addition, the average of December CME Group futures contract price during February is also used as the projected price of the revenue protection in the FCIP in the U.S. Therefore, it is an appropriate measure of price risk. The sample period for the futures prices is from 1980 to 2017. We compare two contracts: *NN72* (the baseline model), and *Linear72* (a linear contract with all 72 weather indices).

Table M.1 summarizes the index insurance performances. We see that after considering price risk, the *NN72* index insurance remains effective in improving farmers' utilities and CEW, stabilizing their wealth distributions, and reducing downside tail risks. The *NN72* contract achieves a CEW improvement of \$19.46/acre in the training sample and \$20.68/acre in the test sample, improving CEW by 4.85% and 5.19% in the training sample and test sample, respectively. This is similar to the case that considers production risk only in Table 4. Comparing *NN72* and *Linear72* contracts, we see that *Linear72* has much worse utility and CEW improvement, even though the premium is similar. Figure M.1 shows that *NN72* achieves a more effective basis risk reduction than *Linear72*.

[27] See Rouwenhorst and Tang (2012) and Kang et al. (2020) for discussions of commodity pricing.

Table M.1: **Protecting both production and price risks.** We consider an index insurance contracts protecting both production and price risks. The *NN72* contract has the 3-hidden-layer (64-64-16 neurons) structure, as in the baseline model. The *Linear72* is a linear contract with all 72 weather indices. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance policies and the CEW improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurer. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses. The sample period is 1980-2017.

Contract	NN72 ($\lambda^* = 1.4137$)		Linear72 ($\lambda^* = 1.3791$)	
Data	Training	Test	Training	Test
Panel A: Utility improvement				
U with insurance	-4.32	-4.37	-4.68	-4.63
U w/o insurance	-5.05	-5.15	-5.05	-5.15
U improvement (%)	14.41%	15.25%	7.47%	10.11%
Panel B: CEW improvement				
CEW with insurance	420.50	419.32	410.76	411.95
CEW w/o insurance	401.05	398.63	401.05	398.63
CEW improvement	19.46	20.68	9.71	13.32
CEW improvement (%)	4.85%	5.19%	2.42%	3.34%
Panel C: Policy characteristics				
Premium	42.28	40.53	41.33	38.15
Coverage	29.91	28.67	29.97	27.66
Insurer Profit	12.37	11.86	11.36	10.49
Panel D: Risk reduction measured by standard deviation				
Std	100.67	102.83	110.20	112.46
Std w/o insurance	132.67	132.65	132.67	132.65
Std reduction	24.12%	22.48%	16.94%	15.22%
Panel E: Risk reduction measured by Value-at-Risk (VaR)				
$\text{VaR}_{5\%}$	333.10	299.72	289.48	288.87
$\text{VaR}_{5\%}$ w/o insurance	256.32	251.00	256.32	251.00
$\text{VaR}_{5\%}$ improvement	76.79	48.71	33.17	37.86



(a) NN72, training set

(b) NN72, test set

(c) Linear72, training set

(d) Linear72, test set

Figure M.1: **Basis risk of index insurance protecting both production and price risks.** These figures plot the insurance payoffs against actual loss, over the training or test set. The index insurance is designed to protect both production and price risks. The *NN72* contract has a 3-hidden-layer (64-64-16 neurons) structure, as in the baseline model. The *Linear72* is a linear contract with 72 weather indices.

N Regulatory costs

The NN-based insurance contract seems to be more complicated than traditional insurances and such contract complexity might increase litigation risks and regulatory frictions. In this section, we further evaluate the impacts of tightening regulatory costs. We quantify the regulatory costs of contract complexity by the regulatory capital reserve. The Solvency II directive has the solvency capital requirement (SCR) to achieve solvency with a 99.5% probability over a one-year horizon. SCR serves as a "soft" supervisory specification, and in practice the actual capital that an insurer has to hold is capped and floored at 50% and 20% of SCR (Towers Watson, 2010). In this section, we test the impacts of supply-side frictions when an insurer reserves regulatory capital amounted to $\{20\%, 35\%, 50\%\}$ of the SCR. We assume the regulatory costs will be priced into the insurance premium, $\pi_e(I)$, as follows,

$$\pi_e(I) := \frac{\lambda}{n}\sum_{j=1}^{n} I(\boldsymbol{x}_j) + \text{Regulatory Capital Holding} \times \text{Cost of Capital}.$$

We assume the insurer's cost of capital is 7%, which is the industry's weighted average cost of capital, according to S&P Global Ratings.

Results are summarized in Table N.1. We see that the farmer's utility and CEW improvements decrease with the regulatory capital holding. But, the index insurance remains effective in utility improvement and basis risk reduction even with the presence of additional regulatory costs.

Table N.1: **Impacts of regulatory costs.** This table presents the impacts of regulatory costs, captured by the regulatory capital reserve of insurers. Columns 2-7 display results for different regulatory capital holding levels as a percentage of solvency capital requirement (20%, 35%, and 50%). The last two columns correspond to our baseline model without regulatory costs. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers (net of regulatory cost). Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR).

Capital Reserving (as % of SCR)	20%		35%		50%		BL	
Data	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement								
U with insurance	-3.58	-3.59	-3.59	-3.61	-3.60	-3.61	-3.57	-3.57
U w/o insurance	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16
U improvement (%)	10.22%	13.85%	9.93%	13.38%	9.60%	13.17%	10.60%	14.35%
Panel B: CEW improvement								
CEW with insurance	444.10	443.90	443.70	443.22	443.25	442.91	444.64	444.61
CEW w/o insurance	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26
CEW improvement	13.47	18.64	13.07	17.96	12.62	17.65	14.00	19.36
CEW improvement (%)	3.13%	4.38%	3.03%	4.22%	2.93%	4.15%	3.25%	4.55%
Panel C: Policy characteristics								
Premium	29.40	29.40	29.73	29.46	30.09	29.83	28.44	28.72
Coverage	23.21	23.22	23.13	22.92	23.08	22.86	22.91	23.13
Insurer Profit	5.60	5.61	5.58	5.53	5.57	5.52	5.53	5.59
Panel D: Risk reduction measured by standard deviation								
Std	53.77	47.71	53.75	48.39	53.82	48.18	54.05	47.49
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	34.38%	39.55%	34.41%	38.68%	34.33%	38.95%	34.04%	39.82%
Panel E: Risk reduction measured by Value-at-Risk (VaR)								
$\text{VaR}_{5\%}$	382.94	376.51	382.39	375.72	381.93	374.85	382.89	379.64
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	66.67	50.60	66.11	49.81	65.65	48.94	66.61	53.73

O Performance of other machine learning models

In this section, we consider an alternative family of machine learning models that could capture high dimensionality and nonlinearity, the regression tree models. Here we compare the performances of NN-based model with different tree-based models, including a simple regression tree, tree bagging, random forest, and tree boosting. Tree "bagging", short for bootstrap aggregation, involves bootstrapping many training samples, building a separate tree model using each training set, and averaging their predictions. Random forest improves over tree bagging by decorrelating the bagged trees. This is achieved by randomly drawing some predictors at each splitting branch. Tree boosting improves the performance of regression tree models by sequentially combining a series of simple, small trees. For detailed introduction of different tree-based models, see Hastie et al. (2009) and Gu et al. (2020). Table O.1 presents the results. The utility improvements in the test set are 5.70%, 6.00%, 8.61%, and 7.95% for single tree, tree bagging, random forest, and tree boosting, respectively. Compared to results in Table 4, the performance of tree-based models is superior than low dimensional linear models (0.55% for *Linear1* and 3.08% for *Linear5*) and polynomial models (3.28% for *Quad5* and 3.84% for *Cubic5*), due to their ability to capture high-dimensionality and non-linearity. However, their performance is worse than NN-based models, with 6%-9% less utility improvement in the test set.

Table O.1: **Tree-based models.** We compare the performance of different tree-based models with our baseline model ($NN72$). Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" represents the baseline case studied in Section 4.2. The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Model	NN72 (BL) ($\lambda^* = 1.2413$)	Single Tree ($\lambda^* = 1.2126$)	Bagging ($\lambda^* = 1.2772$)	Random Forest ($\lambda^* = 1.2766$)	Boosting ($\lambda^* = 1.2554$)
Panel A: Utility improvement					
U with insurance	-3.57	-3.93	-3.91	-3.80	-3.83
U w/o insurance	-4.16	-4.16	-4.16	-4.16	-4.16
U improvement (%)	14.35%	5.70%	6.00%	8.61%	7.95%
Panel B: CEW improvement					
CEW with insurance	444.61	432.59	432.99	436.51	435.61
CEW w/o insurance	425.26	425.26	425.26	425.26	425.26
CEW improvement	19.36	7.34	7.74	11.25	10.35
CEW improvement (%)	4.55%	1.72%	1.82%	2.65%	2.43%
Panel C: Policy characteristics					
Premium	28.72	24.18	17.32	18.59	20.48
Coverage	23.13	19.94	13.56	14.56	16.31
Insurer Profit	5.59	4.24	3.76	4.03	4.17
Panel D: Risk reduction measured by standard deviation					
Std	47.49	69.66	67.66	63.78	64.62
Std w/o insurance	78.92	78.92	78.92	78.92	78.92
Std reduction	39.82%	11.73%	14.27%	19.18%	18.12%
Panel E: Risk reduction measured by Value-at-Risk (VaR)					
$\text{VaR}_{5\%}$	379.64	333.58	337.40	339.78	337.53
$\text{VaR}_{5\%}$ w/o insurance	325.91	325.91	325.91	325.91	325.91
$\text{VaR}_{5\%}$ improvement	53.73	7.67	11.50	13.88	11.63

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