Agricultural Insurance Ratemaking: Development of a New Premium Principle

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Abstract

Determining the appropriate premium to charge for the underlying risk is central to delivering a sustainable agricultural insurance program. While this is fundamental to all types of insurance, in agriculture this is a particularly challenging task given systemic risk, information asymmetry, and a number of multifaceted factors pertaining to the loss experience data, including scarcity and credibility. The objective of this paper is to formally introduce premium principles to the agricultural insurance literature, with a focus on a new premium principle approach based on the multivariate weighted distribution. The multivariate weighted premium principle (MWPP) formalizes the reweighting of historical loss experience using auxiliary factors in order to refine the agricultural insurance pricing. These auxiliary factors may reflect systemic risk and include material information, such as economic and market conditions, weather, soil, etc. In the empirical study, a unique reinsurance data set from the province of Manitoba, Canada, is used to evaluate a number of potential premium principles. With the flexibility of the MWPP, the empirical results indicate that the MWPP approach can be a viable premium principle for pricing agricultural insurance. Furthermore, the MWPP redistributes premium rates and assigns increased loadings to higher risk layers, helping reinsurers manage their reserves and achieve improved sustainability in the long term.

Keywords: Premium principles, Agricultural insurance ratemaking, Systemic risk, Loss reweighting, Weighted distribution

1 Introduction

Agricultural insurance is recognized as one of the most rapidly growing insurance markets (Swiss Re, 2013a). Over the past several decades, it has played an increasingly important role in helping to improve food productivity, achieve food security and protect economic growth (Porth and Tan, 2015; Swiss Re, 2013b). For example, the exposure of the U.S. crop insurance program was approximately USD 117 billion in 2012, with program costs of USD 14.1 billion (Shields, 2013). The respective figures for Canada are CAD 17.3 billion and CAD 1 billion (Agriculture and Agri-Food Canada (AAFC), 2012). The crop insurance sector has experienced rapid growth since 2005, largely due to emerging markets driven by major growth in Brazil, China and India (Swiss Re, 2013a). Figure 1 displays the share of agricultural insurance premiums among different regions. As agricultural insurance programs continue to grow in terms of scale and scope, actuarial foundations have become more important in order to ensure that programs are efficient and actuarially sound. Therefore, central to crop insurance programs worldwide is the ability to determine actuarially fair and sustainable premium rates (Babcock et al., 2004; Woodard, 2014). When premiums are not sound, a number of difficulties can result, including the limited development of agricultural insurance programs, as well as inadequate reinsurance capacity (Coble et al., 2008; Ozaki et al., 2008). Even slight improvements in the accuracy of premium rates can have a considerable impact on reducing government spending and ensuring long-term sustainability of the programs.

Determining the actuarially fair premium rate can be a challenging task, especially in the case of agricultural insurance. This is attributed to a number of challenging issues including systemic risk, moral hazard, adverse selection, as well as data scarcity and credibility

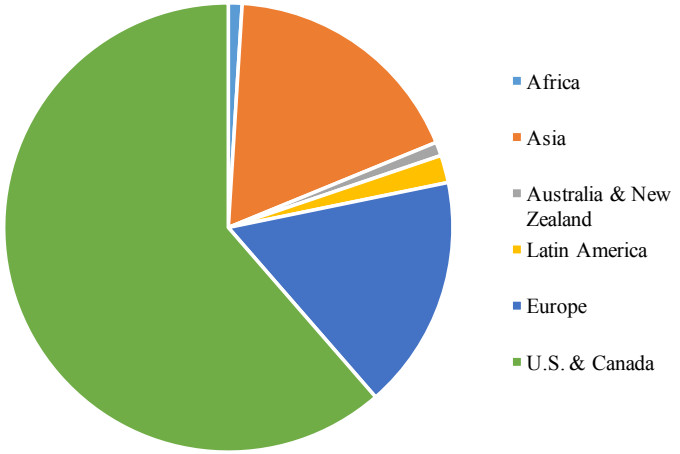


Figure 1: Share of agricultural insurance premiums (Source: Swiss Re, 2013).

(Chambers, 1989; Duncan and Myers, 2000; Glauber, 2004; Goodwin and Ker, 1998; Ker and Goodwin, 2000; Nelson and Loehman, 1987; Skees and Reed, 1986; Woodard and Garcia, 2008). Agriculture is a highly weather-sensitive sector that is largely exposed to increasingly adverse weather conditions (Barnett and Mahul, 2007; Brockett et al., 2009). As a result, systemic risks, such as weather risk, have become a key focus for financial regulators, given these risks can be very difficult to predict and often they have a huge impact on the insurance portfolio (Okhrin et al., 2013; Woodard and Garcia, 2008). As a result, it is imperative to incorporate systemic risk factors into the pricing framework for P&C insurance in general, and agricultural insurance specifically. As well, it is well documented in literature that information asymmetry, including adverse selection and moral hazard, makes pricing crop insurance premiums difficult (Glauber, 2004; Woodard and Garcia, 2008). While information asymmetry is present in all types of insurance, it can be particularly challenging in agriculture in part due to the fact that the loss experience data can be quite scarce and there is often concerns over its' credibility (Porth et al., 2015). For example, in most countries there is only one growing season per year, and hence only one loss observation per year. In countries with developed crop insurance programs where there is a long time series of historical records, this means that approximately 30-40 years of annual historical loss observations can be used for rating products (Porth et al., 2014). However, there is a concern that older loss experience

may not be as relevant today due to program modifications, technological advancements, deviations in farming practices, changes in climate, etc. (Woodard et al., 2012a). Coupled with the fact that extreme agricultural insurance losses, such as floods and droughts, tend to occur relatively infrequently, there is a need to balance using as much of the time series as possible to capture theses significant events, versus the concern that older data may not be credible and, therefore, should be discarded. For most P&C insurance lines the premium is intended to cover the expected loss, plus a loading charge for administration and operating, as well as uncertainty and profit, and possibly other fees, such as product research and development, cost of contingent capital, return on equity, etc. However, most crop insurance programs differ as they are delivered via unique partnership models involving risk-sharing arrangements between government and the private sector. This usually involves the government providing a subsidy for premiums as well as covering the full cost of administration and operating. Therefore, crop insurance premiums are set to cover only the expected loss and a relatively small loading to maintain a reasonable reserve. This means that for most crop insurance programs, such as in the U.S. and Canada, premiums do not include a charge for administration and operating and there is also no provision for profit loadings (Agriculture and Agri-Food Canada (AAFC), 2012; Coble et al., 2013). Over the past decade there has been more attention from government on making crop insurance programs actuarially sound, and this means that rates should be "actuarially fair for all insurance products, in all regions, and ideally for all producers" (Coble et al., 2013).

There is a rich body of literature that has focused on improving crop insurance pricing methods, and to-date most of this research has focused on the U.S. crop insurance system (Babcock et al., 2004; Poon and Lu, 2015; Rejesus et al., 2006; Woodard et al., 2012b; Woodard and Sherrick, 2011). Most crop insurance ratemaking approaches are based on a relatively straightforward method using the simple average of the historical loss cost ratio (LCR), which is defined as the ratio of indemnities to liabilities (Borman et al., 2013). How-

ever, the presence of systemic risks, including extreme weather events and commodity market volatility, for examples, may make the current market approach inadequate and subject to mispricing. Further, the data challenges regarding scarcity and credibility may make the current market practice for pricing limited. To help overcome these data problems, recent research has discussed more comprehensive approaches to pricing in agricultural insurance, including methods to scientifically reweight historical losses from different dimensions to accommodate and adjust for various risk factors. For example, Coble et al. (2011) and Borman et al. (2013) propose to improve pricing through incorporating weights into the data in order to reflect program changes and weather patterns. Woodard (2014) constructs a conditional Weibull distribution model that integrates weather variables and technology evolutions into crop yields explicitly. Porth et al. (2014) propose a "liability weighted" LCR to aggregate historical loss data and introduce a modified credibility model to weight the loss experiences from different geographical regions to enhance the reinsurance pricing model.

This paper contributes to the literature by formalizing the reweighing of historical losses with systemic risk factors using a new class of premium principle, known as the multivariate weighted premium principle (MWPP), in order to improve the actuarial soundness of premiums in the context of agricultural insurance. A key advantage of the MWPP is that it provides a flexible framework to incorporate the weighting of important auxiliary factors. These auxiliary factors may include variables that represent economic and market conditions, weather, soil, etc. Through considering a more comprehensive approach to pricing, it is expected that the incorporation of variables that represent systemic risk will help to overcome the problem of data scarcity and credibility. This idea is motivated by empirical pricing results in agricultural insurance and reinsurance ratemaking, as well as the probability proportional to size sampling method that is widely used in statistical sampling (Patil et al., 1986; Rao, 1965). In this paper, a unique data set obtained from Manitoba Agricultural Services Corporation, which corresponds to the entire crop insurance program in the

province of Manitoba, Canada, is used to empirically examine the proposed MWPP, and compare it to a number of well-known insurance premium principles. The results suggest that incorporating auxiliary information into the crop insurance ratemaking approach can improve the accuracy of pricing for crop insurance. In particular, the proposed MWPP has the property of attaching increased loading to the higher risk reinsurance contract layers, ensuring the insurers and reinsurers to achieve long-term sustainability.

The remainder of this paper is organized as follows. The next section provides an overview of premium principles, including a discussion on desirable properties that a premium principle should possess. Section 3 formally introduces the Multivariate Weighted Premium Principle (MWPP) as well as the multivariate weighted reinsurance pricing formulaes under some specified assumptions. Section 4 empirically analyzes the proposed MWPP using reinsurance data from the province of Manitoba, Canada, including calibration of the premium principles to the reinsurance data and case studies about the impact on pricing reinsurance policies. Section 5 concludes the paper and Appendices collect the proofs.

2 Premium Principles

Premium principle is core to insurance practice in that it provides a way of quantifying an insurable risk X. The insurance risk X is often defined as a non-negative random variable, with its cumulative distribution function $F_X(x)$, survival function $S_X(x)$, and probability density function $f_X(x)$. We denote the collection of all nonnegative random variables as a set $\mathcal{X}$ on the probability space $(\Omega, \mathcal{F}, P)$. Basically, a premium principle is a functional Π assigned to an insurable risk X.

In order for a premium principle to reflect the underlying riskiness of an insurance exposure, it should possess some desirable properties for it to be actuarially sound. The survey paper by Young (2004) extensively discusses these properties, together with an inventory of premium

principles. The key properties of a premium principle can be summarized as follows:

1. Positive risk loading: $\Pi(X) \geq \mathsf{E}(X)$ for all $X \in \mathfrak{X}$. This property requires that the premium charged for insuring the risk is no less than the expected payout.

2. No unjustified risk-loading: For a degenerate risk X, i.e. there exists a constant c such that $\mathsf{P}(X = c) \equiv 1$, then $\Pi(X) = c$. This property implies that if a risk results in a constant loss of c for certain, then the corresponding insurance premium should assign no risk loading.

3. No ripoff: $\Pi(X) \leq \operatorname{ess\,sup}(X)$ for all $X \in \mathfrak{X}$. This property ensures that the insurer should not charge higher than the maximum loss of the risk.

4. Translation invariance: $\Pi(X + a) = \Pi(X) + a$ for all $X \in \mathfrak{X}$ and $a \geq 0$. If a risk X is increased by a constant amount a, then the premium for the combined risk $X + a$ should just be the premium of the original risk plus a.

5. Scale invariance: $\Pi(aX) = a\Pi(X)$ for all $X \in \mathfrak{X}$ and $a \geq 0$. This property is also known as homogeneity of degree one in economic literature, and its significance is to avoid arbitrage opportunities. For example, the premium for $2X$ should correspond to the premium for two insurance policies for the risk X, otherwise, there is a possibility of arbitrage.

   Combining Property 4 and Property 5 implies linearity.

6. Subadditivity: $\Pi(X + Y) \leq \Pi(X) + \Pi(Y)$ for all $X \in \mathfrak{X}$ and $Y \in \mathfrak{X}$. Subadditivity requires that insuring a portfolio of homogeneous risk should be less expensive than insuring individual risks separately because of the risk pooling and diversification benefit. This property implies that the insurer cannot benefit from dividing risk into pieces.

7. First stochastic dominance (FSD) preserving: If $S_X(x) \leq S_Y(x)$ for all $x \geq 0$, then $\Pi(X) \leq \Pi(Y)$.

8. Stop-loss (SL) ordering preserving: If $\mathsf{E}\big[(X-d)_+\big] \leq \mathsf{E}\big[(Y-d)_+\big]$ for all $d>0$, then $\Pi(X) \leq \Pi(Y)$. Here $(x)_+ = \max(x,0)$.

While there are a large number of premium principles, some commonly used ones include the following:

a. Expectation Premium Principle: $\Pi^e(X) = (1+\theta)\mathsf{E}(X)$, where $\theta > 0$. This is the most widely used premium principle in agricultural insurance ratemaking, as well as in other insurance applications due to its simplicity.

b. Standard Deviation Premium Principle: $\Pi^{sd}(X) = \mathsf{E}(X) + \theta\sqrt{\mathsf{Var}(X)}$, where $\theta > 0$ and $\mathsf{Var}(X)$ is the variance of the random variable X. This premium principle incorporates a risk loading that is proportional to the standard deviation of the insured risk. While widely used in general P&C insurance, this premium principle has received little attention in agricultural insurance, with the exception of the work by Porth et al. (2013), which empirically analyzed the optimal reinsurance contract structure for Manitoba Agricultural Services Corporation (MASC) in the province of Manitoba, Canada.

c. Esscher Premium Principle: $\Pi^{ess} = \dfrac{\mathsf{E}(Xe^{\theta X})}{\mathsf{E}(e^{\theta X})}$, where $\theta > 0$. This premium principle, which is based on *Esscher Transform*, is widely used in option pricing in the context of incomplete markets (E.g. Bühlmann et al., 1996; Gerber and Shiu, 1994; Hubalek and Sgarra, 2006). It is interesting to note that this premium principle is a special case of the *Equilibrium Premium Principle* proposed by Bühlmann (1980). Additionally, a more general form of the Esscher premium principle is referred to as the *Exponential Tilting Premium Principle* (Heilmann, 1989; Kamps, 1998).

d. Distortion Premium Principle: For any increasing concave function $g : [0,1] \mapsto [0,1]$ with $g(0) = 0, g(1) = 1$, the premium is calculated as $\Pi^d(X) = \int_0^\infty g\big(S_X(u)\big)du$ (Wang, 1996; Wang et al., 1997). The function g is called the *distortion* function and $g\big(S_X(u)\big)$

is called the distorted probability. A popular special case of this premium class is called *Proportional Hazards Premium Principle* (Wang, 1995).

3 Multivariate Weighted Premium Principle (MWPP)

3.1 Auxiliary Factors and MWPP

Fundamental to a sustainable insurance program is charging an appropriate premium for the underlying risk. By attaching a premium to each risk, insurers are essentially ordering the relative riskiness of the risks (Wang, 1995). Therefore, most existing premium principles focus only on the properties of X. In insurance practice, however, it is likely that premiums not only depend on the risk itself, but, also on many other factors such as general economic conditions, the insurer's and reinsurer's capital positions, etc. (Bühlmann, 1980; Deprez and Gerber, 1985; Furman and Zitikis, 2009). For example, 2013 was a challenging year for Canadian property and casualty (P&C) insurers who experienced historically high levels of catastrophic activity (MARSH, 2014). Therefore, it is reasonable to expect that extreme events, such as floods, fires and hurricanes, may have significant impacts on the P&C insurance and reinsurance loss profiles.

As a special sector of P&C insurance, agricultural insurance losses are highly variable and often geographically correlated due to possible climate changes and adverse weather conditions (Porth et al., 2015; 2014). As pointed out in the introduction that pricing crop insurance can be especially challenging due to systemic risk and difficulties related to the historical loss data, including scarcity and credibility. In recent years, agricultural insurance research has focused on ratemaking methodologies to weight historical losses in order to incorporate weather risk and preserve the credibility of the data (Borman et al., 2013; Coble et al., 2011; Porth et al., 2014; Woodard, 2014). This paper contributes to this line of inquiry by

providing a new approach to weight the historical loss experience by integrating auxiliary factors into the pricing framework. This is achieved by proposing a new class of premium principles, known as the Multivariate Weighted Premium Principle (MWPP).

The idea of weighted distributions has been discussed in a number of research papers. In statistics, the concept of weighted distributions was formulated and developed by Rao (1965), where weighted distributions are used to model the probability of recording observations. Furman and Zitikis (2008a) formulate a premium principle based on the univariate weighted distribution, which has the advantage of unifying many popular premium principles. They further extend this weighting idea beyond the conditional state independence by introducing another non-negative random variable $Y \in \mathcal{X}$, which is associated with X, into the pricing functional $\Pi : \mathcal{X} \times \mathcal{X} \to [0, \infty)$. In particular, from an asset allocation point of view, the resulting economic weighted pricing functional Π incorporates a number of risk capital allocation rules, such as those based on modified tail covariance and exponential tilting (Furman and Landsman, 2006; 2008; Furman and Zitikis, 2008c; 2010; Wang, 2007). The MWPP is based on the multivariate weighted distribution and is formally defined as follows:

Definition 3.1. *Consider a random loss X, the multivariate weighted premium with respect to a given positive random vector $\boldsymbol{Y}$ and a weighting function $w : [0, \infty] \mapsto [0, \infty]$, with $\mathsf{Cov}\big(X, w(\boldsymbol{Y})\big) \geq 0$, is defined as*

$$\Pi^w(X, \boldsymbol{Y}) \;=\; \frac{\mathsf{E}\big(w(\boldsymbol{Y})X\big)}{\mathsf{E}\big(w(\boldsymbol{Y})\big)}. \tag{1}$$

Note that the MWPP is a general premium principle that depends not only on the underlying risk X, but also the auxiliary factors $\boldsymbol{Y}$ and the weighting function w. The MWPP provides a way to scientifically integrate additional information provided by $\boldsymbol{Y}$ and w, in order to produces a premium principle that is actuarially sound. We have the following two additional remarks:

Remark 3.1. *MWPP is defined based on the weighted density transform:* $\mathfrak{T}^w : (X, \boldsymbol{Y}) \to X^w$*, associated with the random vector* $\boldsymbol{Y}$ *and the weighting function* w*. The MWPP is the expectation of the resulting weighted random variable* X^w *as* $\Pi^w(X, \boldsymbol{Y}) = \mathsf{E}(\mathfrak{T}^w) = \frac{\mathsf{E}\left(w(\boldsymbol{Y})X\right)}{\mathsf{E}\left(w(\boldsymbol{Y})\right)}$.

Remark 3.2. *It should also be emphasized that* $\boldsymbol{Y}$ *is a random vector of auxiliary risk factors that needs not be different from* X*. In fact. When* $\boldsymbol{Y} = X$*, the multivariate weighted premium principle in Definition 3.1 degenerates to the weighted premium discussed in Furman and Zitikis (2008b). As a result, MWPP adds flexibility to the premium principle, including other existing premium principles as special cases, depending on the choices of* $\boldsymbol{Y}$ *and* w*. Besides the univariate weighted premium in Furman and Zitikis (2008b), the Esscher premium and the Equilibrium Premium Principle are also special cases of MWPP.*

As outlined in the previous section, there are eight desirable properties that a premium principle should satisfy. Zhu et al. (2016) provide a formal analysis of MWPP and demonstrate that this premium principle indeed satisfies all of the eight desirable properties. Table 1 compares and contrasts the properties of MWPP and the four premium principles discussed earlier. A check mark ("✓") indicates that the premium principle satisfies the corresponding property, otherwise we use a cross-mark ("✗"). Note that the distortion premium principle is the only other premium principle that also satisfies all eight properties.

3.2 Pricing Agricultural Insurance Policy

Let us consider a type of layer contract widely found in P&C insurance. For predefined constants a and b, a layer $(a, b]$ of loss, denoted by $\mathbf{L}_{[a,b]}$, is defined as follows:

$$\mathbf{L}_{[a,b]} = \begin{cases} 0, & X < a \\ X - a, & a \le X < b \\ b - a, & X \ge b. \end{cases} \tag{2}$$

Table 1: Properties of different premium principles. "Expectation" is for Expectation Premium Principle, "SD" is for Standard Deviation Premium Principle, "Esscher" is for Esscher Premium Principle, "Distortion" is for Distortion Premium Principle, and "MWPP" is for Multivariate Weighted Premium Principle.

Property	a. Expectation	b. SD	c. Esscher	d. Distortion	e. MWPP
1. Risk loading	✓	✓	✓	✓	✓[a]
2. No unjustified	✗	✓	✓	✓	✓
3. No ripoff	✗	✗	✓	✓	✓
4. Translation	✗	✓	✓	✓	✓
5. Scale	✓	✓	✗	✓	✓[b]
6. Subadditivity	✓	✗	✗	✓	✓[c]
7. FSD	✓	✗	✗	✓	✓[d]
8. SL	✓	✗	✗	✓	✓[e]

[a] See Proposition 3.1 in Zhu et al. (2016).
[b] X and $\boldsymbol{Y}$ are different random variables.
[c] X and $\boldsymbol{Y}$ are different random variables.
[d] See Proposition 3.2 in Zhu et al. (2016).
[e] See Corollary 3.1 in Zhu et al. (2016).

A layer contract therefore covers part of the losses between layers a (the retention) and b (the limit), with the maximum payout limited to the difference $b - a$. It is also called the excess of loss (XoL) insurance policy. The private reinsurance program in Canada is based on a modified version of the XoL insurance policy, which covers a certain percentage of the standard layer policy. More specifically, the reinsurance contract has the form $\tau \mathbf{L}_{[a,b]}$, where $0 < \tau < 1$ is the coverage level.

To provide some explicit formulaes for pricing the above XoL reinsurance policy using the MWPP, we need to provide additional specifications with respect to the auxiliary factors $\boldsymbol{Y}$ and the weighting function w. For simplicity, we assume that $\boldsymbol{Y} = (Y_1, \ldots, Y_k)^T$ is k-th variate normally distributed with mean vector $\boldsymbol{\mu_Y} = (\mu_1, \ldots, \mu_k)^T$ and covariance matrix $\boldsymbol{\Sigma_Y}$; i.e. $\boldsymbol{Y} \sim N_k(\boldsymbol{\mu_Y}, \boldsymbol{\Sigma_Y})$. We also assume X and $\boldsymbol{Y}$ follow the multivariate normal

distribution; i.e. $(\boldsymbol{Y}, X) \sim N_{k+1}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ where

$$\begin{aligned} \boldsymbol{\mu} &= (\boldsymbol{\mu_Y}, \mu_x)^T = (\mu_1, \ldots, \mu_k, \mu_x)^T, \\ \boldsymbol{\Sigma} &= \begin{pmatrix} \boldsymbol{\Sigma_Y}, & \boldsymbol{\sigma_{YX}} \\ \boldsymbol{\sigma_{YX}^T} & \sigma_x^2 \end{pmatrix}. \end{aligned}$$

We now turn to the weight function w. While there could be many choices of w, for simplicity we consider the following two types of the weighting function:

- **Linear weighting function:**

$$w_l(y) = \alpha_0 + \boldsymbol{\alpha}^T \boldsymbol{y}; \tag{3}$$

- **Exponential weighting function:**

$$w_e(y) = e^{\alpha_0 + \boldsymbol{\alpha}^T \boldsymbol{y}}; \tag{4}$$

where $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_k)^T, \boldsymbol{y} = (y_1, \ldots, y_k)^T$.

Given the above setup, the multivariate weighted reinsurance pricing formulae can now be stated. These results are collected in Proposition 3.1 and Proposition 3.2 for weights that are linear and exponential, respectively.

Proposition 3.1. *(***Linear Weight Premium Principle***) Under MWPP, and together with the assumptions* $(\boldsymbol{Y}, X) \sim N_{k+1}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ *and the weighting function* $w_l(y)$ *in Equation* (3)*, the premium* $\Pi^{w_l}(X, Y)$ *for the reinsurance policy* $\tau \mathbf{L}_{[a,b]}$ *is given by*

$$\begin{aligned} &\Pi^{w_l}(X, Y) \\ &= \big[\mu_x - a + \frac{\sigma_x^2 \beta_1}{\alpha_0 + \boldsymbol{\alpha}^T \boldsymbol{\mu_Y}}\big]\big[\Phi(b_1) - \Phi(a_1)\big] + (b-a)\big[1 - \Phi(b_1)\big] + \sigma_x\big[\phi(a_1) - \phi(b_1)\big], \end{aligned} \tag{5}$$

where

$$a_1 = \frac{a-\mu_x}{\sigma_x}, \qquad b_1 = \frac{b-\mu_x}{\sigma_x}, \tag{6}$$

$$\beta_0 = \alpha_0 + \boldsymbol{\alpha}^T \boldsymbol{\mu_Y} - \frac{\boldsymbol{\alpha}^T \boldsymbol{\sigma_{YX}}}{\sigma_x^2}\mu_x. \tag{7}$$

$$\beta_1 = \frac{\boldsymbol{\alpha}^T \boldsymbol{\sigma_{YX}}}{\sigma_x^2}, \tag{8}$$

$\Phi(\cdot)$ *and* $\phi(\cdot)$ *represent the cumulative distribution function and the density function of the standard normal distribution, respectively.*

Proposition 3.2. *(***Exponential Weight Premium Principle***) Under MWPP, and together with the assumptions* $(\boldsymbol{Y}, X) \sim N_{k+1}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ *and the weighting function* $w_e(y)$ *in Equation* (4)*, the premium* $\Pi^{w_e}(X, Y)$ *for the reinsurance policy* $\tau \mathbf{L}_{[a,b]}$ *is given by*

$$\Pi^{w_e}(X,Y) = \sigma_x\big[\phi(a_2) - \phi(b_2)\big] + (\mu_x - a + \beta_1\sigma_x^2)\big[\Phi(b_2) - \Phi(a_2)\big] + (b-a)\big[1 - \Phi(b_2)\big], \tag{9}$$

where

$$a_2 = \frac{a-\mu_x-\beta_1\sigma_x^2}{\sigma_x}, \qquad b_2 = \frac{b-\mu_x-\beta_1\sigma_x^2}{\sigma_x}, \tag{10}$$

and β_1 *is defined according to Equation* (8)*.*

Proofs of Propositions 3.1 and 3.2 are relegated to Appendix A. The formulas for other premium principles (including Expectation Premium Principle, Standard Deviation Premium Principle, Esscher's Premium Principle and Distortion Premium Principle) are discussed in Appendix B.

It should be emphasized that the normality assumption in the above two propositions is chosen for tractability. Other joint distribution can be assumed and the tractability of the resulting MWPP pricing formula depends on the complexity of the assumed joint distribu-

tion. In the event that explicit pricing formula is not available, the insurance premium can be approximated numerically. Furman and Zitikis (2008c) propose to split the covariance of $\mathsf{Cov}(X, \boldsymbol{Y})$. Zhu et al. (2016) also discuss this issue under a copula framework.

4 Empirical Analysis

In this section, the MWPP is empirically analyzed using a unique data set representing the reinsurance program in the province of Manitoba, Canada, over the period from 2001 to 2011. Subsection 4.1 calibrates the various premium principles to the given data set. Subsection 4.2 compares the impact of various premium principles on pricing different layers of risk. Finally, a sustainability study is provided in Subsection 4.3.

One of the key features of MWPP is its flexibility for incorporating many desirable auxiliary factors and weight functions. However, it is important to only choose those factors with market data that are sufficiently available in order to facilitate calibration, and also to keep the number of factors to a reasonable size to ensure model parsimony. For this reason, in this paper a MWPP with two types of auxiliary factors are considered, including weather and economic variables. Agriculture is a very weather sensitive sector, with some research suggests that 70-90% of agriculture loss is due to adverse weather (Motha, 2011; O'Connor, 2013). This means that the agricultural insurance and reinsurance markets are exposed to systemic weather risks, such as droughts or floods that can cause many farms within a region to experience substantial crop losses. For example, it is well documented in literature that temperature, as measured by growing degree days (GDD), have a strong relationship between agricultural yields and losses (Kaufmann and Snell, 1997; Prentice et al., 1992; Schlenker et al., 2006). In addition, weather derivatives are written on accumulated growing degree

days (AGDD) in the market. GDD in a single day t is defined as follows:

$$GDD_t = \max(0, T_t - \widetilde{T}), \tag{11}$$

where T_t is the daily average temperature on day t; $\widetilde{T}$ is the reference temperature and is usually set to be 65 $^\circ F$ (about 18 $^\circ C$) in the lituerature. Economic variables are also promising candidates for auxiliary factors since economic conditions may indirectly impact the agriculture sector (Fischer et al., 2005; Mendelsohn et al., 1994), and hence be an important consideration in ratemaking. There are many economic variables to consider as economic auxiliary factors. For example, gross domestic product (GDP), consumer price index (CPI) and retailer price index (RPI) are all good candidates. Given that they are highly correlated economic variables, for the purpose of illustration we select one year lag Canadian GDP as the second auxiliary factor. Table 2 provides summary statistics of the historical losses and auxiliary factors. The data has skewness and kurtosis values that are very close to zero, implying that multivariate normal is an appropriate model for their joint distribution. To confirm this observation, a number of univariate and multivariate normality tests are also conducted, with details of the testing results shown in Appendix C. The testing results reassure that none of the univariate and multivariate normality tests are rejected, and hence there is no strong evidence against the multivariate normality assumption.

4.1 Calibrating the Premium Principles

The objective of this subsection is to calibrate previously described premium principles to the private reinsurance data set. We furnish the calibration via the least squares method by minimizing the sum of the squared residuals, where residuals are calculated as the difference between each premium principle and the actual reinsurance premiums. The explicit MWPP XoL pricing formulae for the linear weighted (3) and exponential weighted functions (4)

Table 2: Summary statistics for the historical loss experience for the reinsurance program in Manitoba, Canada, as well as for the two auxiliary factors corresponding to growing season GDD and the Canadian GDP. The data period for reinsurance loss experience and GDD is from 2001 to 2011. The data period for GDP is from 2000 to 2010.

Summary Statistics			
	Loss (CA$ million)	GDD ($^{\circ}C$)	GDP (CA$ billion)
Mean	147.91	89.59	1392.91
Standard Deviation	100.99	36.59	207.73
Skewness	0.76	-0.47	-0.10
Kurtosis	-0.49	-0.42	-1.62

Data Source: temperature data is from the Environment Canada and GDP is from International Monetary Fund (IMF). The data period for reinsurance loss experience and GDD is from 2001 to 2011. The data period for GDP is from 2000 to 2010.

(see Propositions 3.1 and 3.2, respectively) and the corresponding pricing formulae for other premium principles (see Appendix B) greatly enhance the calibration. The results are listed in Table 3. The estimated parameters for the linear weighted and the exponential weighted premium principle are significant at the 0.01 level, providing support in favour of introducing auxiliary factors into the pricing framework. It is also of interest to note that for the distortion premium principle, a is estimated to be 1. Therefore, the Beta distortion function degenerates to the dual power distortion function, and the same estimating result can be obtained using the dual power distortion function (See Appendix B). This result suggests that the dual power distortion function is more suitable for the data studied in this paper compared to the PH transform.

4.2 The Impact on Pricing

In this section, the properties and effects of the premium principles on the reinsurance policies are evaluated, based on the calibrated premium premium principle in Subsection 4.1. The analysis is divided into two parts: (i) Absolute risk loading and (ii) Relative risk loading.

Table 3: Estimating results for different premium principles using the least square method. Standard errors are in the parentheses below the corresponding estimated coefficient values. "***" means significant at 0.01 level.

Premium Principles	Parameter	Value
Expectation	θ	0.0722^{***}
		(0.0054)
Standard Deviation	θ	0.1753^{***}
		(0.0007)
Escheer's ($\times 0.01$)	θ	0.1963^{***}
		(0.0032)
Distortion	a	1.0000^{***}
		(0.1798)
	b	1.1492^{***}
		(0.4468)
Linear Weighted	α_0	0.0021^{***}
		(0.0000)
	α_1	1.9426^{***}
		(0.0001)
	α_2	1.0316^{***}
		(0.0016)
Exponeqntial Weighted	α_0	5.7823^{***}
		(0.3619)
	α_1	2.8900^{***}
		(0.0000)
	α_2	3.3471^{***}
		(0.1299)

(i) Absolute Risk Loading (ARL)

In the first part of the analysis, the relative effects of the premium principles on pricing different layers of risk are considered. The premium allocated for each layer is defined as the absolute risk loading (ARL). More specifically,

$$\text{ARL}(x) \quad = \quad \Pi(\mathbf{L}_{(x,x+h]}), \quad h > 0, \tag{12}$$

The premium principle Π is said to have decreasing absolute risk load if $\text{ARL}(x)$ is decreasing in x. Decreasing absolute risk load is attractive because it preserves the stochastic order among layers (Kaas et al., 1994; Wang, 1995). Zhu et al. (2016) shows in theory that the MWPP has decreasing ARL. In this section, we demonstrate this property by pricing a series of XoL contracts $\mathbf{L}_{[a,a+h)}$, where a increases from 0 to 550, in increments of 50. We also set $h = 50$, except for the last layer which in this case we have $a = 550$ and $h = \infty$. Hence, the maximum payout for all layers, except the last one, is capped at 50 [1].

The pricing results are displayed in Table 4 and Figure 2. The first observation is that these results violate the "no ripoff" property of both the expectation and standard deviation premium principles. For example, the prices of $\mathbf{L}_{[0,50)}$ under both the expectation and the standard deviation premium principles exceed the maximum payout of 50. This observation suggests that there is an unjust penalty when these premium principles are used, especially for pricing lower layers of risk. On the other hand, the pricing results also show that these premium principles are not appropriate for pricing layers of extreme risk. More specifically, the prices of $\mathbf{L}_{[550,\infty)}$ are 0.0209 and 0.1952 for the expectation and standard deviation premium principles, respectively. The expectation premium principle produces the lowest price for this layer of risk compared to all of the premium principles. However, at the same time, the expectation premium principle severely penalizes risk in the lower layers.

[1] The unit of contract layers is million USD. These layers are selected according to empirical reinsurance data

In contrast, the standard deviation premium principle grossly overestimates the tail risk, producing a price that is several multiples of the other premium principles. Together these observations suggest that the expectation and standard deviation premium principles may not be actuarially sound when used for crop reinsurance pricing.

In comparing the remaining four premium principles, it is found that the Esscher premium principle produces the lowest premium and the exponential weighted premium principle produces the highest premium over all the considered layers of risk. Furthermore the pricing results from both the distortion and the linear weighted premium principles are very similar to each other. The results from both the linear and exponential weighted premium principles indicate that any desirable loading can be constructed by the appropriate choices of auxiliary factors and the weight function.

Table 4: Absolute risk load of different contract layers under each premium principle. Each row shows the layer of the contract, while the columns show the results for different premium principles. "Expectation" stands for Expectation Premium Principle, "SD" represents Standard Deviation Premium Principle, "Esscher" stands for Esscher's Premium Principle, "Distortion" is for Distortion Premium Principle, "L-Weighted" means Linear Weighted Premium Principle, and "E-Weighted" stands for Exponential Weighted Premium Principle.

Contract	**Expectation**	**SD**	**Esscher**	**Distortion**	**L-Weighted**	**E-Weighted**
$[0, 50)	52.3616	50.0203	48.9200	49.3281	49.2125	49.7790
$[50, 100)	50.0255	48.6513	46.9001	47.7516	47.5456	49.1664
$[100, 150)	45.2246	45.1156	42.7122	44.0454	43.8243	47.4631
$[150, 200)	37.4290	38.6477	35.7847	37.3487	37.3018	43.7217
$[200, 250)	27.4260	29.6621	26.6419	28.0297	28.3169	37.2284
$[250, 300)	17.2827	19.9288	17.0605	18.0145	18.5819	28.3238
$[300, 350)	9.1545	11.5845	9.1485	9.6738	10.2807	18.6737
$[350, 400)	4.0074	5.8421	4.0314	4.2689	4.7072	10.4089
$[400, 450)	1.4321	2.6058	1.4431	1.5315	1.7600	4.8155
$[450, 500)	0.4141	1.0629	0.4167	0.4436	0.5322	1.8242
$[500, 550)	0.0963	0.4102	0.0966	0.1032	0.1293	0.5604
$[550, \infty)	0.0209	0.1952	0.0216	0.0224	0.0295	0.1711

(ii) Relative risk loading (RRL)

Agricultural reinsurers are particularly concerned with catastrophic losses due to the significant exposures they face in regards to systemic risk, such as weather risks and spatially

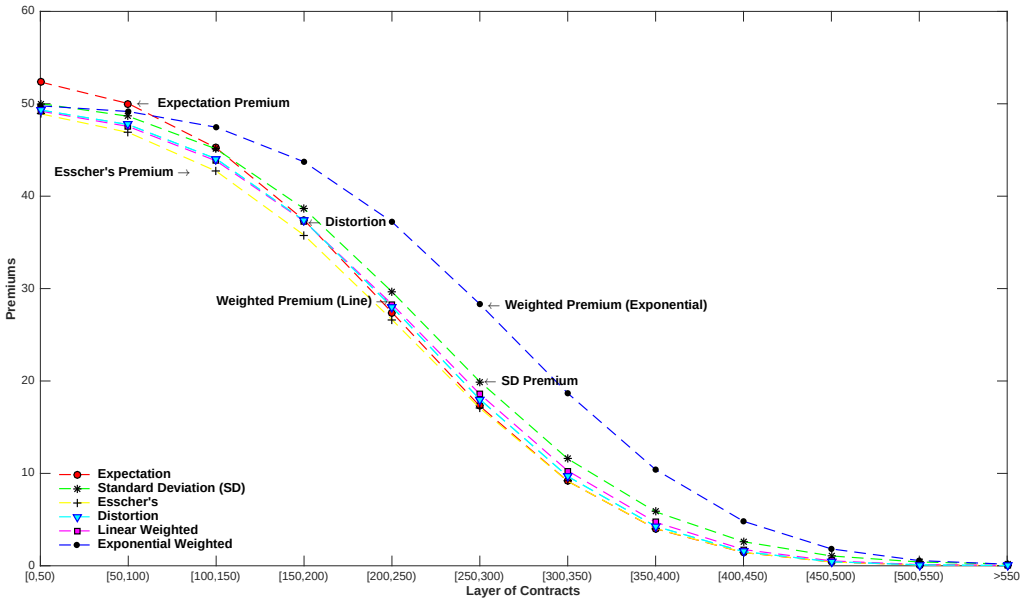


Figure 2: Absolute risk load for different layers of risk under each premium principle. (Premiums and contract layers are all in million USD.)

correlated geographical risks. Therefore, the adopted premium principle for agricultural insurance and reinsurance purposes should have the appropriate risk loading, particularly for the higher risk layers. A very useful measure for quantifying the risk loading in the extremum is to examine the relative risk loading (RRL) of a given premium principle[2]. Mathematically, RRL is defined as

$$\mathrm{RRL}(x) \quad = \quad \lim_{h \to 0} \frac{\Pi(\mathbf{L}_{(x,x+h]})}{\mathsf{E}(\mathbf{L}_{(x,x+h]})}. \tag{13}$$

The interpretation of this definition is straightforward. In general, for any premium principle the premium for the risk X, $\Pi(X)$, can always be written as

$$\Pi(X) \quad = \quad \mathsf{E}(X) + \Theta(X), \tag{14}$$

[2]This relates to the notion of Marginal Indemnity Functions (MIF), which measures the increase in ceded loss per unit of increase in the globle loss (For more details, see: Assa, 2015; Zhuang et al., 2016). It is interesting to note that the layer contract discussed in this paper (indemnity as defined in Equaion (2)), is the optimal reinsurance solution for the ceding company with the MIF being either 0 or 1.

where $\Theta(X)$ is the risk loading corresponding to risk X. For example, $\Theta(X) = \theta\mathsf{E}(X)$ for the Expectation Premium Principle and $\Theta(X) = \theta\sqrt{\mathsf{Var}(X)}$ for the Standard Deviation Premium Principle. Therefore, $\text{RRL} - 1 = \lim_{h\to 0} \dfrac{\Theta(\mathbf{L}_{(x,x+h]})}{\mathsf{E}(\mathbf{L}_{(x,x+h]})}$ compares the risk loading at each contract layer relative to the net premium. Higher RRL indicates greater risk-adjusted loading to the layer relative to the net premium.

To better understand the behavior of the RRL among the different premium principles, Figure 3 plots the RRL over various values of x by setting h to a very small value of $h = 5$ (instead of taking the limit to zero). The Expectation Premium Principle has a constant RRL of 1.0722. This is not surprising since the calibrated θ is 0.722 (see Table 3). The figure also reveals that the RRL of the Esscher's premium remains stable at a value close to 1 for all contract layers. In fact, it can be formally shown that in the limit as $h \mapsto 0$, the RRL of Esscher's premium converges to 1. This property is asserted in the following proposition with the proof found in Appendix D.

Proposition 4.1. *The Esscher's premium principle,* $\Pi^{ess} = \dfrac{E\left(Xe^{\theta X}\right)}{E\left(e^{\theta X}\right)}$, *has a constant relative risk loading; i.e.,*

$$RRL(x) \quad = \quad \lim_{h\to 0} \frac{\Pi^{ess}(\mathbf{L}_{(x,x+h]})}{E(\mathbf{L}_{(x,x+h]})} = 1. \tag{15}$$

In view of Proposition 4.1, the Esscher's Premium Principle has the same shortcoming as the expectation premium principle in the sense that both do not produce higher risk loadings for higher layers of risk. Comparing the RRL of the Standard Deviation Premium Principle (RRL_{SD}) and the Exponential Weighted Premium Principle ($\text{RRL}_{\text{E-Weighted}}$), we find that at lower layers $\text{RRL}_{\text{SD}} > \text{RRL}_{\text{E-Weighted}}$, while $\text{RRL}_{\text{SD}} < \text{RRL}_{\text{E-Weighted}}$ for higher layers with more tail exposure. Therefore, the Exponential Weighted Premium Principle is able to assign higher RRL at tail layers compared to the other premium principles.

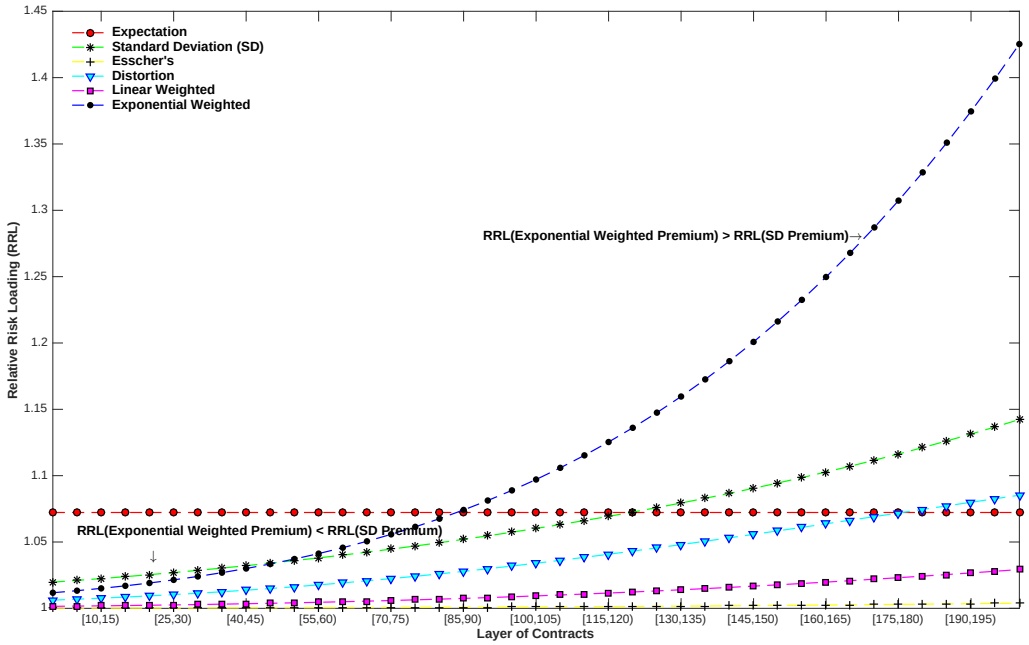


Figure 3: Relative risk loading for different contract layers under each premium principle. (Contract layers are all in million USD.)

4.3 Sustainability and Recovery Analysis

In this section, we present two case studies to provide additional insight on the impact of the premium principles on the long term sustainability of a reinsurer's book of business. For the first case study, we assume that the reinsurer begins with a zero account value and writes the XoL contracts $\mathbf{L}_{[a,b]}$, with one year maturity, and four hypothetical pairs contract layers (in million USD): $[0, 100]$, $[100, 200]$, $[200, 300]$, and $[300, 400]$. These four XoL contracts are labelled A, B, C, and D, respectively. The same XoL contract is renewed at the beginning of each year and the premium is charged according to the adopted premium principle. At the beginning of each year, the net account value (i.e. premium received plus the accumulated account value less indemnities), if positive, is invested in the equity market. The equity return is assumed to follow a lognormal process with annual parameters $\mu_S = 0.05$ and $\sigma_S = 0.2$. If the net account value is negative, i.e. the business is in deficit, the reinsurer

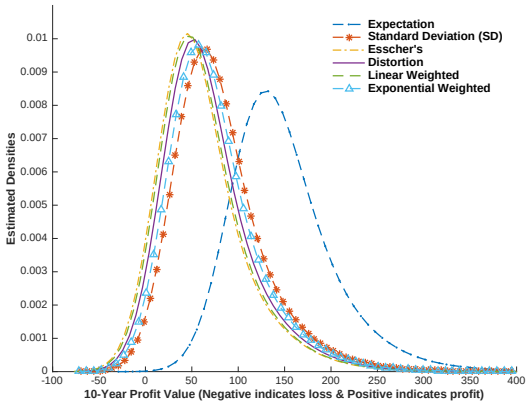
is permitted to borrow from a bank at the risk-free rate $r_f = 0.9\%$. With the above setup, the reinsurer is interested in its business's profit and loss (P&L) profile at the end of 10 years.

The summary statistics corresponding to each XoL contract and premium principle are depicted in Table 5. These results are calculated based on 100,000 simulated scenarios. Figure 4 provides a graphical comparison of the corresponding simulated distributions of the reinsurer's 10-year profits and losses. In addition to the mean (Mean), standard deviation (SD), coefficient of variation (CV), skewness (Skewness), and kurtosis (Kurtosis), Table 5 also produces three other measures useful for quantifying the simulated profit distribution. These are the Value-at-Risk (VaR), the conditional tail expectation (CTE), and the probability of default (Pr(default)). Care should be given to interpreting these values since positive values represent profit and negative values signify loss. The reported VaR corresponds to the 95% percentile of the profit distribution, or equivalently, 95% percentile of the loss distribution. On the other hand, CTE quantifies "expected shortfall" in that it captures how bad the reinsurer's average loss can get if the loss falls below its VaR. In this context a profit distribution with a larger VaR or CTE is preferred. Among 100,000 simulated scenarios, "Pr(default)" gives the percentage of these scenarios with negative account value at the end of 10 years.

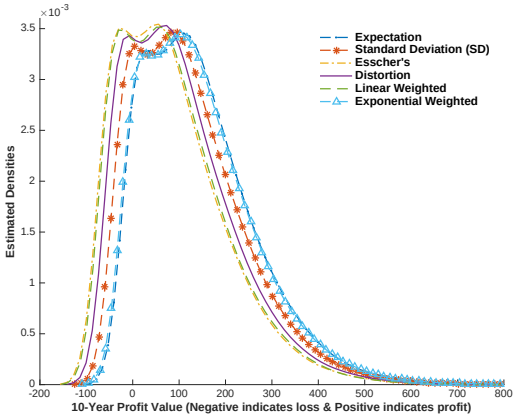
The importance of the pricing assumption on the underlying XoL contract is clearly demonstrated in our simulation results. For example, the premium calculated based on the expectation premium principle appears to be overly conservative for the least risky contract (i.e. Contract A). This is consistent with our earlier observation from Figures 2 and 3. For higher layers of XoL, such as Contracts C and D, the expectation premium principle seems inadequate to capture the higher exposed tail risk, as exemplified by the higher standard deviations. The VaR, CTE, and Pr(default) are also the worst among all the premium principles for Contract D. On the other hand, the weighted premium principles have the

Table 5: Summary statistics of the P&L simulation results (Unit: Million USD$). Results are based on 100,000 simulation of iterations.

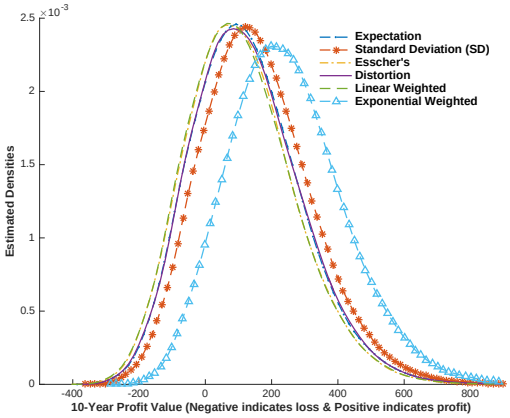
Contract A: [0, 100]								
Premium Principles	**Mean**	**Std**	**CV**	**Skew**	**Kurt**	**VaR**	**CTE**	**Pr(default)**
Expectation	148.07	81.59	0.33	1.64	7.60	91.36	83.74	0.00
SD	79.73	63.17	0.53	1.95	8.73	37.08	32.51	0.00
Esscher	60.47	58.15	0.65	2.01	8.62	22.75	19.30	0.00
Distort	67.34	59.92	0.60	2.04	9.06	27.72	23.88	0.00
WeightLinear	62.62	59.29	0.64	2.10	9.53	24.47	20.90	0.00
WeightExponential	73.33	61.79	0.57	1.98	8.97	31.95	27.74	0.00
Contract B: [100, 200]								
Premium Principles	**Mean**	**Std**	**CV**	**Skew**	**Kurt**	**VaR**	**CTE**	**Pr(default)**
Expectation	143.68	118.92	0.83	1.08	4.66	-3.87	-7.59	0.08
SD	120.51	118.97	0.99	1.10	4.73	-25.97	-29.68	0.17
Esscher	82.18	114.93	1.40	1.10	4.71	-58.41	-62.05	0.28
Distort	99.83	115.65	1.16	1.08	4.61	-42.61	-46.31	0.22
WeightLinear	85.93	115.44	1.34	1.11	4.85	-55.36	-58.94	0.27
WeightExponential	141.19	120.15	0.85	1.10	4.77	-8.24	-12.64	0.10
Contract C: [200, 300]								
Premium Principles	**Mean**	**Std**	**CV**	**Skew**	**Kurt**	**VaR**	**CTE**	**Pr(default)**
Expectation	124.83	163.19	1.31	0.48	3.40	-119.68	-166.49	0.23
SD	155.59	166.23	1.07	0.48	3.40	-93.79	-142.29	0.18
Esscher	108.75	162.05	1.49	0.47	3.36	-134.81	-181.07	0.26
Distort	125.37	164.23	1.31	0.48	3.36	-119.64	-167.88	0.23
WeightLinear	107.87	163.02	1.51	0.49	3.40	-135.68	-182.76	0.27
WeightExponential	245.31	174.75	0.71	0.52	3.52	-18.03	-65.61	0.07
Contract D: [300, 400]								
Premium Principles	**Mean**	**Std**	**CV**	**Skew**	**Kurt**	**VaR**	**CTE**	**Pr(default)**
Expectation	93.18	142.61	1.53	-0.01	2.96	-141.92	-200.71	0.26
SD	147.68	146.70	0.99	0.03	2.98	-92.03	-151.72	0.16
Esscher	102.65	142.86	1.39	0.00	2.95	-132.60	-190.52	0.24
Distort	106.60	143.64	1.35	0.00	2.98	-129.18	-188.63	0.23
WeightLinear	100.48	143.14	1.42	0.00	2.96	-134.43	-193.70	0.24
WeightExponential	258.50	154.09	0.60	0.08	3.10	6.13	-54.07	0.05



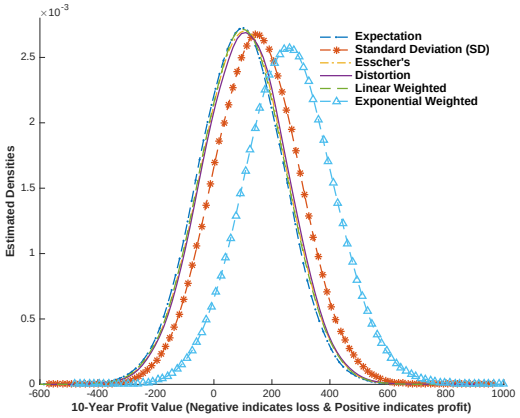
(a) Contract A: [0, 100].



(b) Contract B: [100, 200].



(c) Contract C: [200, 300].



(d) Contract D: [300, 400].

Figure 4: 10-year P&L distribution for each contract under different premium principles.

advantage of assigning higher risk loadings to riskier contracts, resulting in lower risk measures and lower default probabilities, particularly for the exponentially weighted premium principle. This also explains why the simulated profit and loss distribution under the exponentially weighted premium principle is on the right, relative to other simulated densities (see Figure 4). One may of course criticize that the exponentially weighted premium principle excessively penalizes tail risk.

We now proceed to the second case study. In the previous case study the analysis was based on writing new reinsurance business with zero initial capital. However, in this case study we assume that the reinsurer is already in financial distress with an initial financial position in deficit. The magnitude of the deficit corresponds to the 95% VaR of its P&L assuming the reinsurer has been writing yearly renewable XoL contracts $\mathbf{L}_{[a,b]}$ with $a = 336$, $b = 617$ (in million USD) for the last 10 years with its premium based on the Esscher premium principle. This deficit can be computed from the preceding case study. The selected reinsurance layers are consistent with the market practice. Given that the insurer is in financial distress, it is exploring various premium strategies in order to recover from its deficit position. This analysis, in turn, provides useful insight to its long term sustainability.

We assume that the reinsurer continues to write the same yearly renewable XoL contracts except with a different premium principle. For each premium strategy, we compute the following performance measures from 100,000 simulated scenarios:

- Average recovery time (in years) reverting from deficit position to surplus position;
- 80 percentile of the average recover time;
- probability of default.

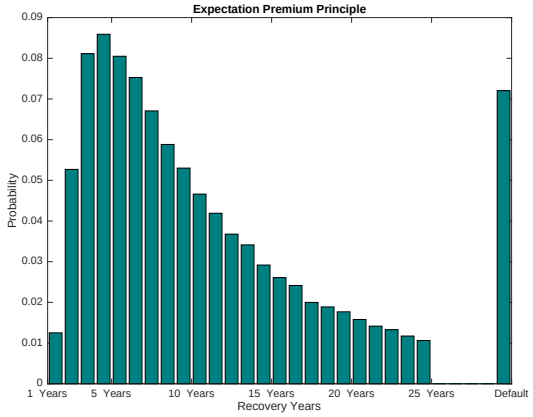
The results are reported in Table 6 along with the histograms of recovery time displayed in Figure 5. Note that these values are calculated assuming the reinsurer can finance its deficit by continually borrowing at the risk-free rate until a maximum grace period of 25 years. If

the reinsurer is declared defaulted if it is still in deficit after 25 years.

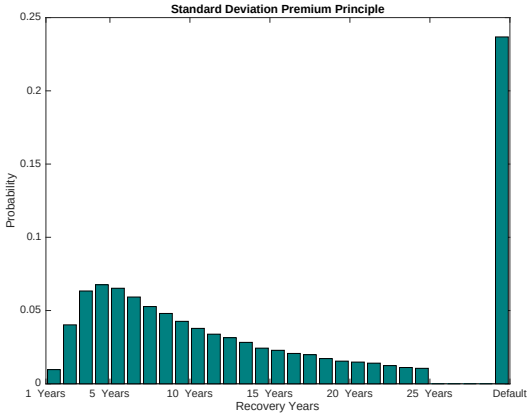
If the reinsurer continues to price the reinsurance portfolio using the Esscher premium principle, on average it will take 7.9 years to recover from the deficit and its default probability is 11.89%. If the reinsurer modifies its premium strategy, its recovery time increases to as high as 9.23 years (with expectation premium prinicple) or decreases to as low as 1.89 years (with MWPP). Similarly the probability of default of 11.9% also increases to 14.64% for the expectation premium principle or decreases to 0.01% for the MWPP. The histograms in Figure 5 also show that with the exception of the exponential weighted premium principle, the other premium principles produce a spike at "default", indicating the company could still be in deficit after 25 years. Together these results highlight the importance of the premium principle on the reinsurer's deficit recovery. By strategically changing premium principles, such as that based on the exponential weighted, it is possible to produce a long term sustainable business solution, especially in the presence of extreme adverse weather events, which is a major systemic risk in underwriting agriculture business.

Table 6: Summary of recovery analysis. For each premium principle, the first row shows the average recovery time; the second row shows 80% recovery time; the last row shows default probabilities. "Expectation" stands for Expectation Premium Principle, "SD" represents Standard Deviation Premium Principle, "Esscher" stands for Esscher's Premium Principle, "Distortion" is for Distortion Premium Principle, "L-Weighted" means Linear Weighted Premium Principle, and "E-Weighted" stands for Exponential Weighted Premium Principle. Results are based on 100,000 simulation of iterations.

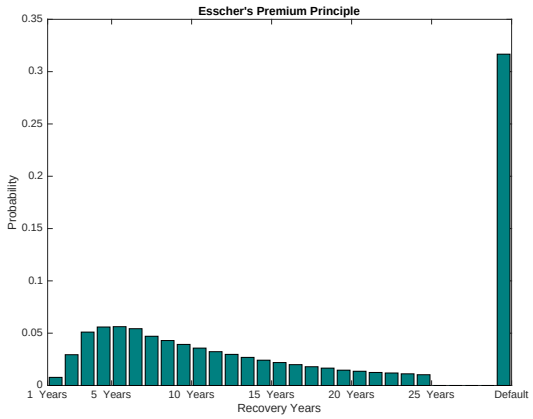
	Expectation	**SD**	**Esscher**	**Distortion**	**L-Weighted**	**E-Weighted**
Avg. Recovery	9.2391	7.4413	7.8696	8.8658	8.3857	1.8876
80% Recovery	16	10	11	14	13	3
Default Prob.	0.1464	0.1031	0.1189	0.1487	0.1365	0.0001



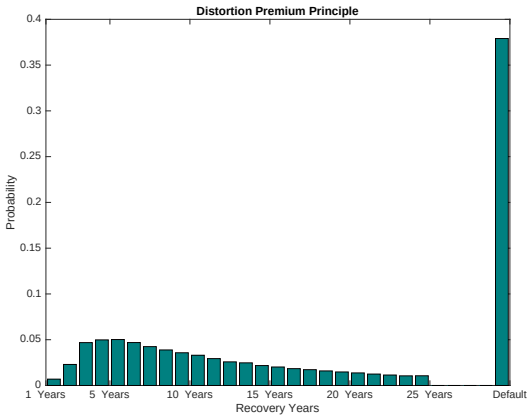
(a) Expectation Premium Principle.



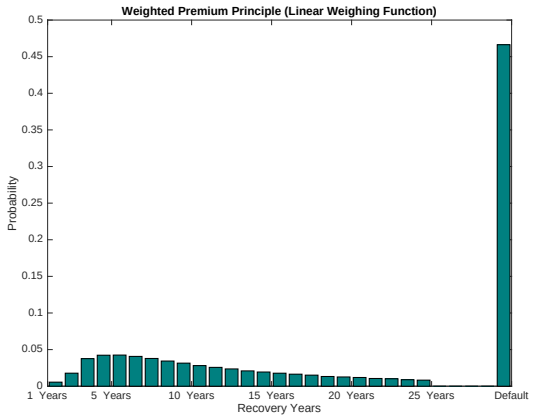
(b) Standard Deviation Premium Principle.



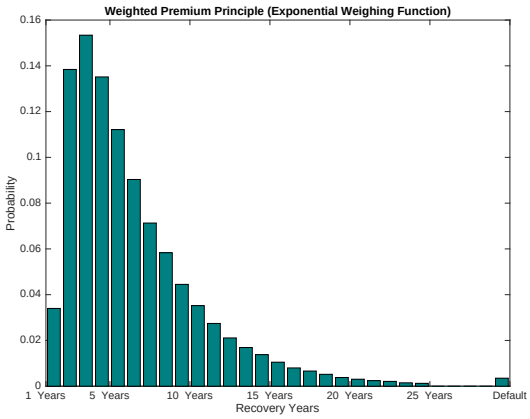
(c) Esscher's Premium Principle.



(d) Distortion Premium Principle.



(e) Weighted Premium Principle (Linear).



(f) Weighted Premium Principle (Exponential).

Figure 5: 25-year recovery times under different premium principles.

5 Conclusion

Agricultural insurance pricing is challenging due to a number of unique features compared to most other commercial lines of P&C insurance. Some of these difficulties include systemic risk, information asymmetry, shortness of data, and other structural factors of the loss experience. Therefore, an important topic in agricultural insurance is how to determine the appropriate premium. This paper addresses this issue and for the first time formally introduces premium principles to the agricultural insurance and reinsurance literature. The focus is on a new premium principle approach based on the multivariate weighted distribution, which reweights historical loss experience using auxiliary factors, which may include economic and market conditions, weather, soil, etc., as examples. In the empirical analysis, two variables are selected, including weather as measured by Growing Degree Days (GDD) as well as one year lagged Gross Domestic Product (GDP). A unique data set representing the entire crop insurance program in Manitoba, Canada is used to empirically examine the new MWPP. The results suggest that the MWPP approach produces the most accurate premium rates. In particular, the MWPP redistributes premium rates and assigns increased loadings to higher risk contract layers, helping reinsurers to manage their reserves and achieve improved sustainability in the long term.

A Proof of Proposition 3.1

First we define the indicator function $\mathbb{I}_A$ as

$$\mathbb{I}_A = \begin{cases} 1, & x \in A, \\ 0, & x \notin A. \end{cases} \tag{16}$$

Since $\boldsymbol{Y} \sim N(\boldsymbol{\mu_Y}, \boldsymbol{\Sigma_Y})$, then

$$\mathsf{E}(w_l(\boldsymbol{Y})) \;=\; \alpha_0 + \boldsymbol{\alpha}^T \boldsymbol{\mu_Y}. \tag{17}$$

Also note that $\mathsf{E}(w_l(\boldsymbol{Y})\mathbf{L}_{[a,b]}) = \mathsf{E}(w_l(\boldsymbol{Y})(X-a)\mathbb{I}_{\{a\le X<b\}}) + \mathsf{E}(w_l(\boldsymbol{Y})(b-a)\mathbb{I}_{\{X\ge b\}})$, we consider the two parts separately. To proceed, let us first consider the distribution of $\boldsymbol{Y}$ conditional on $X = x$. It can be shown that $\boldsymbol{Y}|X = x \sim N(\breve{\mu}, \breve{\Sigma})$, where $\breve{\mu} = \boldsymbol{\mu_Y} + \frac{\boldsymbol{\sigma_{YX}}}{\sigma_x^2}(x-\mu_x)$ and $\breve{\Sigma} = \boldsymbol{\Sigma_Y} - \frac{\boldsymbol{\sigma_{YX}}\boldsymbol{\sigma_{YX}}^T}{\sigma_x^2}$. Therefore, $\alpha_0 + \boldsymbol{\alpha}^T Y|X = x \sim N(\widetilde{\mu}, \widetilde{\Sigma})$, where $\widetilde{\mu} = \alpha_0 + \boldsymbol{\alpha}^T\breve{\mu}, \widetilde{\Sigma} = \boldsymbol{\alpha}^T\breve{\Sigma}\boldsymbol{\alpha}$. For notation simplicity, let us define a_1, b_1, β_0 and β_1 according to Equations (6), (7) and (8). Therefore, $\widetilde{\mu} = \beta_0 + \beta_1 x$, and

$$\begin{aligned}
\mathsf{E}(w_l(Y)(X-a)\mathbb{I}_{\{a\le X<b\}}) &= \mathsf{E}\big((X-a)\mathbb{I}_{\{a\le X<b\}}\mathsf{E}(w_l(Y)|X))\big) \\
&= \mathsf{E}\big((\beta_0+\beta_1 X)(X-a)\mathbb{I}_{\{a\le X<b\}}\big) \\
&= (\beta_0 + a\beta_1)\mathsf{E}\big((X-a)\mathbb{I}_{\{a\le X<b\}}\big) + \beta_1\mathsf{E}\big((X-a)^2\mathbb{I}_{\{a\le X<b\}}\big).
\end{aligned}$$

Also note that,

$$\begin{aligned}
\mathsf{E}\big((X-a)\mathbb{I}_{\{a\le X<b\}}\big) &= \int_a^b (x-a)\frac{1}{\sqrt{2\pi}\sigma_x}e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}dx \\
&= \sigma_x[\phi\big(a_1\big) - \phi\big(b_1\big)] + (\mu_x - a)[\Phi\big(b_1\big) - \Phi\big(a_1\big)]. \qquad (18) \\
\mathsf{E}\big((X-a)^2\mathbb{I}_{\{a\le X<b\}}\big) &= \int_a^b x^2\frac{1}{\sqrt{2\pi}\sigma_x}e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}dx - 2a\int_a^b x\frac{1}{\sqrt{2\pi}\sigma_x}e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}dx \\
&\quad + a^2\int_a^b \frac{1}{\sqrt{2\pi}\sigma_x}e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}dx.
\end{aligned}$$

Specially,

$$\begin{aligned}\int_a^b x^2 \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} dx &= (\mu_x^2+\sigma_x^2)\big\{\Phi\big(b_1\big)-\Phi\big(a_1\big)\big\} \\ &\quad +\sigma_x(a+\mu_x)\phi\big(a_1\big)-\sigma_x(b+\mu_x)\phi\big(b_1\big), \qquad (19)\\ \int_a^b x \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} dx &= \mu_x\big\{\Phi\big(b_1\big)-\Phi\big(a_1\big)\big\}-\sigma_x\big\{\phi\big(b_1\big)-\phi\big(a_1\big)\big\}. \qquad (20)\end{aligned}$$

Therefore,

$$\begin{aligned}\mathsf{E}\big((X-a)^2\mathbb{I}_{\{a\le X<b\}}\big) &= \big((\mu_x-a)^2+\sigma_x^2\big)\big\{\Phi\big(b_1\big)-\Phi\big(a_1\big)\big\} \\ &\quad +\phi\big(b_1\big)(2a-b-\mu_x)\sigma_x+\phi\big(a_1\big)(\mu_x-a)\sigma_x, \qquad (21)\end{aligned}$$

Combing Equations (18) and (21), we have, Therefore,

$$\begin{aligned}\mathsf{E}(w_l(Y)\mathbf{L}_{[a,b]}) &= \sigma_x(\beta_0+\beta_1\mu_x)\big\{\phi\big(a_1\big)-\phi\big(b_1\big)\big\} \\ &\quad +\big((\mu_x-a)(\beta_0+\beta_1\mu_x)+\sigma_x^2\beta_1\big)\big[\Phi\big(b_1\big)-\Phi\big(a_1\big)\big] \\ &\quad +(b-a)(\beta_0+\beta_1\mu_x)\big[1-\Phi\big(b_1\big)\big]. \qquad (22)\end{aligned}$$

Also note that $\beta_0+\beta_1\mu_x=\alpha_0+\boldsymbol{\alpha}^T\boldsymbol{\mu_Y}$, hence combining Equations (17) and (22), we can show that Equation (5) holds.

A.1 Proof of Proposition 3.2

Since $\boldsymbol{\alpha}^T Y \sim N(\boldsymbol{\alpha}^T\boldsymbol{\mu_Y}, \boldsymbol{\alpha}^T\boldsymbol{\Sigma_Y}\boldsymbol{\alpha})$, then

$$\mathsf{E}\big(w_e(Y)\big) = \mathsf{E}\big(e^{\alpha_0+\boldsymbol{\alpha}^T Y}\big) = \exp\big(\alpha_0+\boldsymbol{\alpha}^T\boldsymbol{\mu_Y}+\frac{1}{2}\boldsymbol{\alpha}^T\boldsymbol{\Sigma_Y}\boldsymbol{\alpha}\big). \qquad (23)$$

Additionally, from the proof of Proposition 3.1, we have

$$\mathsf{E}\big(w_e(Y)|X\big) = \exp\big(\beta_0 + \frac{1}{2}\widetilde{\Sigma} + \beta_1 X\big) = \kappa e^{\beta_1 X},$$

where β_0 and β_1 are defined in Equations (7) and (8), and

$$\begin{aligned}\widetilde{\Sigma} &= \boldsymbol{\alpha}^T\left(\boldsymbol{\Sigma_Y} - \frac{\boldsymbol{\sigma_{YX}}\boldsymbol{\sigma_{YX}}^T}{\sigma_x^2}\right)\boldsymbol{\alpha}\\ \kappa &= \exp\big(\beta_0 + \frac{1}{2}\widetilde{\Sigma}\big).\end{aligned}$$

Specially, consider $\int_a^b (x-a)e^{\beta_1(x-a)}\frac{1}{\sqrt{2\pi}\sigma_x}e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}dx =$

$$\begin{aligned}&\int_a^b (x-\mu_x)e^{\beta_1(x-a)}\frac{1}{\sqrt{2\pi}\sigma_x}e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}dx + \int_a^b (\mu_x - a)e^{\beta_1(x-a)}\frac{1}{\sqrt{2\pi}\sigma_x}e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}dx\\
\overset{y=\frac{x-\mu_x}{\sigma_x}}{=}\ & e^{\beta_1(\mu_x-a)}\int_{\frac{a-\mu_x}{\sigma_x}}^{\frac{b-\mu_x}{\sigma_x}}\big(\sigma_x(y-\sigma_x\beta_1) + (\mu_x - a + \beta_1\sigma_x^2)\big)\frac{1}{\sqrt{2\pi}}e^{-\frac{(y-\beta_1\sigma_x)^2}{2}+\frac{1}{2}\beta_1^2\sigma_x^2}dy\\
=\ & \sigma_x e^{\beta_1(\mu_x-a)+\frac{1}{2}\beta_1^2\sigma_x^2}\int_{a_1}^{b_1} v\frac{1}{\sqrt{2\pi}}e^{-\frac{v^2}{2}}dv + (\mu_x - a + \beta_1\sigma_x^2)e^{\beta_1(\mu_x-a)+\frac{1}{2}\beta_1^2\sigma_x^2}\int_{a_1}^{b_1}\frac{1}{\sqrt{2\pi}}e^{-\frac{v^2}{2}}dv\\
& \text{where}\quad v = y - \beta_1\sigma_x,\ a_2 = \frac{a-\mu_x-\beta_1\sigma_x^2}{\sigma_x}\ \text{and}\ b_2 = \frac{b-\mu_x-\beta_1\sigma_x^2}{\sigma_x},\\
=\ & e^{\beta_1(\mu_x-a)+\frac{1}{2}\beta_1^2\sigma_x^2}\big[\sigma_x\big(\phi(a_2)-\phi(b_2)\big) + (\mu_x - a + \beta_1\sigma_x^2)\big(\Phi(b_2)-\Phi(a_2)\big)\big], \qquad (24)\end{aligned}$$

Also note that $\mathsf{E}\big(w_e(Y)\mathbf{L}_{[a,b]}\big) = \mathsf{E}\big(w_e(y)(X-a)\mathbb{I}_{\{a\le X<b\}}\big) + \mathsf{E}\big(w_e(y)(b-a)\mathbb{I}_{\{X\ge b\}}\big)$, we have

$$\begin{aligned}\mathsf{E}\big(w_e(Y)(X-a)\mathbb{I}_{\{a\le X,b\}}\big) &= \kappa e^{\beta_1\mu_x+\frac{1}{2}\beta_1^2\sigma_x^2}\big[\sigma_x\big(\phi(a_2)-\phi(b_2)\big) + (\mu_x - a + \beta_1\sigma_x^2)\big(\Phi(b_2)-\Phi(a_2)\big)\big]\\
\mathsf{E}\big(w_e(Y)(b-a)\mathbb{I}_{\{X\ge b\}}\big) &= \kappa(b-a)e^{\beta_1\mu_x+\frac{1}{2}\beta_1^2\sigma_x^2}(1-\Phi(b_2)),\end{aligned}$$

and hence,

$$\begin{aligned}\mathsf{E}\big(w_e(Y)\mathbf{L}_{[a,b]}\big) &= \kappa e^{\beta_1\mu_x+\frac{1}{2}\beta_1^2\sigma_x^2}\big\{\sigma_x\big[\phi\big(a_2\big)-\phi\big(b_2\big)\big]+(b-a)\big[1-\Phi\big(b_2\big)\big]\\ &\quad +(\mu_x-a+\beta_1\sigma_x^2)\big[\Phi\big(b_2\big)-\Phi\big(a_2\big)\big]\big\}. \end{aligned} \tag{25}$$

Also note that $\kappa e^{\beta_1\mu_x+\frac{1}{2}\beta_1^2\sigma_x^2}=\mathsf{E}\big(w_e(Y)\big)$, therefore, Equation (9) holds.

B Formulas for Other Premium Principles

We provide closed form presentations of reinsurance premiums for the XoL reinsurance policy defined in Equation (2) under some other premium principles.

Expectation Premium Principle

Under the expectation premium principle; i.e., $\Pi^e=(1+\theta)\mathsf{E}(\tau\mathbf{L}_{[a,b]})$, the reinsurance premium can be calculated according to Proposition B.1.

Proposition B.1. *Under the expectation premium principle, the premium* Π^e *for the reinsurance contract* $\tau\mathbf{L}_{[a,b]}$ *is given by*

$$\Pi^e=\tau(1+\theta)\big[(b-a)+(\mu_x-b)\Phi(b_1)-\sigma_x\phi(b_1)-(\mu_x-a)\Phi(a_1)+\sigma_x\phi(b_1)\big], \tag{26}$$

where a_1 *and* b_1 *are defined in Equation* (6).

Proof. $\mathsf{E}(\mathbf{L}_{[a,b]})=\mathsf{E}\big((X-a)\mathbb{I}_{\{a\le X<b\}}\big)+\mathsf{E}\big((b-a)\mathbb{I}_{\{X\ge b\}}\big)$, and $X\sim N(\mu_x,\sigma_x^2)$. Also note that $\mathsf{E}\big((X-a)\mathbb{I}_{\{a\le X<b\}}\big)$ can be calculated according to Equation (18), and

$$\mathsf{E}\big((b-a)\mathbb{I}_{\{X\ge b\}}\big) = (b-a)[1-\Phi\big(b_1\big)]. \tag{27}$$

$$\begin{aligned}\mathsf{E}(\mathbf{L}_{[a,b]}) &= (b-a)+(\mu_x-b)\Phi\big(b_1\big)-\sigma_x\phi\big(b_1\big)\\ &\quad -(\mu_x-a)\Phi\big(a_1\big)+\sigma_x\phi\big(a_1\big).\end{aligned} \tag{28}$$

Therefore, the reinsurance premium under the expectation premium principle is

$$s\Pi^e = \tau(1+\theta)\big[(b-a) + (\mu_x - b)\Phi(b_1) - \sigma_x\phi(b_1) - (\mu_x - a)\Phi(a_1) + \sigma_x\phi(b_1)\big]. \tag{29}$$

□

Standard Deviation Premium Principle

For the standard deviation premium principle, i.e., $\Pi^{sd} = \mathsf{E}(\tau\mathbf{L}_{[a,b]}) + \theta\sqrt{\mathsf{Var}(\tau\mathbf{L}_{[a,b]})}$, the reinsurance premium can be calculated according to Proposition B.2.

Proposition B.2. *Under the standard deviation premium principle, the premium* Π^{sd} *for the reinsurance contract* $\tau\mathbf{L}_{[a,b]}$ *is given by*

$$\Pi^{sd} \;=\; \tau L_1 + \tau\theta\sqrt{L_2 - (L_1)^2}, \tag{30}$$

where a_1, b_1 *are defined according to Equation (6), and*

$$\begin{aligned} L_1 &= E(\mathbf{L}_{[a,b]}) \\ &= (b-a) + (\mu_x - b)\Phi(b_1) - \sigma_x\phi(b_1) - (\mu_x - a)\Phi(a_1) + \sigma_x\phi(a_1). \qquad (31) \\ L_2 &= E\big((\mathbf{L}_{[a,b]})^2\big) \\ &= \sigma_x(\mu_x - a)\phi(a_1) - \big((\mu_x - a)^2 + \sigma_x^2\big)\Phi(a_1) + (b-a)^2 \\ &\quad + \big((\mu_x - a)^2 + \sigma_x^2 - (b-a)^2\big)\Phi(b_1) + \sigma_x(2a - b - \mu_x)\phi(b_1). \qquad (32) \end{aligned}$$

Proof. Note that $\mathsf{E}\big((\mathbf{L}_{[a,b]})^2\big) = \mathsf{E}\big((X-a)^2\mathbb{I}_{\{a\le X<b\}}\big) + \mathsf{E}\big((b-a)^2\mathbb{I}_{\{X\ge b\}}\big)$, and $\mathsf{E}\big((X-a)^2\mathbb{I}_{\{a\le X<b\}}\big)$ can be calculated according to Equation (21). Hence, $\mathsf{E}\big((\mathbf{L}_{[a,b]})^2\big)$ can be calculated according to Equation (32). For notation simplicity, let us denote $L_1 = \mathsf{E}(\mathbf{L}_{[a,b]})$ and $L_2 = \mathsf{E}\big((\mathbf{L}_{[a,b]})^2\big)$, then $\mathsf{Var}(\mathbf{L}_{[a,b]}) = L_2 - (L_1)^2$. Combining Equations (28) and (32), we can see that Equation (30) holds. □

Esscher's Premium Principle

For the Esscher's premium principle; i.e., $\Pi^{ess} = \tau \frac{\mathsf{E}\big(\exp(\theta\tau \mathbf{L}_{[a,b]})\mathbf{L}_{[a,b]}\big)}{\mathsf{E}\big(\exp(\theta\tau \mathbf{L}_{[a,b]})\big)}$, the reinsurance premium can be computed according to Proposition B.3.

Proposition B.3. *Under the expectation premium principle, the premium Π^{ess} for the reinsurance contract $\tau\mathbf{L}_{[a,b]}$ is given by*

$$\Pi^{ess} = \frac{c_1\{\sigma_x[\phi(a_2) - \phi(b_2)] + (\mu_x - a + \theta_\tau\sigma_x^2)[\Phi(b_2) - \Phi(a_2)]\} + (b-a)c_2[1-\Phi(b_1)]}{c_1[\Phi(b_2) - \Phi(a_2)] + \Phi(a_1) + c_2[1-\Phi(b_1)]} \tag{33}$$

where a_1, b_1 are defined according to Equations (6), $\theta_\tau = \tau\theta$, and

$$a_3 = \frac{a - \mu_x - \theta_\tau\sigma_x^2}{\sigma_x}, \qquad b_3 = \frac{b - \mu_x - \theta_\tau\sigma_x^2}{\sigma_x} \tag{34}$$

$$c_1 = e^{\theta_\tau(\mu_x - a) + \frac{1}{2}\theta_\tau^2\sigma_x^2}, \qquad c_2 = e^{\theta_\tau(b-a)} \tag{35}$$

Proof. Recall that

$$\Pi^{ess} = \tau \frac{\mathsf{E}\big(\exp(\theta_\tau \mathbf{L}_{[a,b]})\mathbf{L}_{[a,b]}\big)}{\mathsf{E}\big(\exp(\theta_\tau \mathbf{L}_{[a,b]})\big)}, \tag{36}$$

where $\theta_\tau = \theta\tau$. We calculate the two parts in the fraction separately.

$$\begin{aligned}
\mathsf{E}\big(\exp(\theta_\tau \mathbf{L}_{[a,b]})\mathbf{L}_{[a,b]}\big) &= \int_a^b (x-a)e^{\theta_\tau(x-a)} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} dx + (b-a)e^{\theta_\tau(b-a)}\mathsf{E}\big(\mathbb{I}_{\{X\geq b\}}\big), \\
&= e^{\theta_\tau(\mu_x - a) + \frac{1}{2}\theta_\tau^2\sigma_x^2}\big\{\sigma_x\big[\phi\big(a_3\big) - \phi\big(b_3\big)\big] + (\mu_x - a + \theta_\tau\sigma_x^2)\big[\Phi\big(b_3\big) - \Phi\big(a_3\big)\big]\big\} \\
&\quad + (b-a)e^{\theta_\tau(b-a)}\big[1 - \Phi\big(b_1\big)\big], \qquad (37) \\
\mathsf{E}\big(\exp(\theta_\tau \mathbf{L}_{[a,b]})\big) &= \mathsf{E}\big(\mathbb{I}_{\{X<a\}}\big) + \int_a^b e^{\theta_\tau(x-a)} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} dx + e^{\theta_\tau(b-a)}\mathsf{E}\big(\mathbb{I}_{\{X\geq b\}}\big). \\
&= e^{\theta_\tau(\mu_x - a) + \frac{1}{2}\theta_\tau^2\sigma_x^2}\big[\Phi\big(b_3\big) - \Phi\big(a_3\big)\big] + \Phi\big(a_1\big) + e^{\theta_\tau(b-a)}\big[1 - \Phi\big(b_1\big)\big]. \qquad (38)
\end{aligned}$$

Combining Equations (36), (37) and (38), and defining $a_1, b_1, a_2, b_2, c_1, c_2$ according to Equations (6), (10), and (35), we can get the expression of the premium as in Equation (33). □

Distortion Premium Principle

Recall that the distortion premium principle is defined as $\Pi^d = \int_0^\infty g\big(S_{\mathbf{L}_{[a,b]}}(u)\big)du$. Also note that the survival function of the random variable, $\mathbf{L}_{[a,b]}$, can be written as

$$S_{\mathbf{L}_{[a,b]}}(u) = \begin{cases} S_x(u+a) & 0 \leq u < b-a, \\ 0 & u \geq b-a, \end{cases} \tag{39}$$

therefore,

$$\Pi^d = \int_a^b g\big(S_X(u)\big)du. \tag{40}$$

For the distortion function $g(x)$, we consider a general class of distortion functions called Beta transform, defined as

$$g_{a,b}(u) \;=\; \beta(a,b;u) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\int_0^u t^{a-1}(1-t)^{b-1}dt. \tag{41}$$

One advantage of Beta distortion family, with the form of the incomplete beta function (Hogg and Klugman, 1984) is that it includes two important distortion functions as special cases: PH transform and Dual power transform. More specifically, if we set $a = 1$, beta transform degenerates to dual power transform with $\theta = b$:

$$g_\theta(u) \;=\; 1-(1-u)^\theta, \quad \theta \geq 1; \tag{42}$$

while if we set $b = 1$, Beta distortion function degenerates to PH transform with $\theta = 1/a$:

$$g_\theta(u) \;=\; u^{\frac{1}{\theta}}, \quad \theta \geq 1. \tag{43}$$

Wirch and Hardy (1999) consider the beta distortion family in the context of distortion risk measures and discuss the advantage of utilizing the whole loss distribution rather than focusing entirely on the tail, such as conditional tail expectation, which is important for capital adequacy.

C Statistical Validation of Auxiliary Vectors

According to the statistical summary in Table 2, the data has skewness and kurtosis values that are very close to zero, implying that multivariate normal is an appropriate model for their joint distribution. To confirm this observation, a number of univariate and multivariate normality tests are conducted. The univariate normality tests considered in this study include Shapiro-Wilk's test, Cramer-von Mises's test, Kolmogorov-Smirnov's test, and Anderson-Darling's test, and the multivariate normality include Mardia's test, Henze-Zirkler's test and Royston's test. [3] The test results are listed in Table 7 and a "✔" signifies that the data passes the test at the 0.05 significant level. It is reassuring that none of the univariate and multivariate normality tests are rejected, and hence there is no strong evidence against the normality assumption. It is therefore reasonable to have the following model specification:

$$\begin{aligned} X &= \mu_x + \epsilon_x, \quad \text{where} \quad \epsilon_x \sim N(0, \sigma_x^2), \\ Y_{\text{GDD}} &= \mu_{\text{GDD}} + \epsilon_{\text{GDD}}, \quad \text{where} \quad \epsilon_{\text{GDD}} \sim N(0, \sigma_{\text{GDD}}^2), \\ Y_{\text{GDP}} &= \mu_{\text{GDP}} + \epsilon_{\text{GDP}}, \quad \text{where} \quad \epsilon_{\text{GDP}} \sim N(0, \sigma_{\text{GDP}}^2), \end{aligned}$$

and $(X, Y_{\text{GDD}}, Y_{\text{GDP}})$ is jointly normally distributed with means $(\mu_x, \mu_{\text{GDD}}, \mu_{\text{GDP}})$ and variances $(\sigma_x^2, \sigma_{\text{GDD}}^2, \sigma_{\text{GDP}}^2)$.

[3] For a more detailed introduction of these tests, refer to Anderson and Darling (1952), Anderson (1962), Henzea and Zirkler (1990), Mardia (1970), Royston (1991), Shapiro and Wilk (1965), and Smirnov (1948).

Table 7: Normality test results, including univariate normality tests and multivariate normality tests. A "✓" is added if the test is significant at the 0.05 significant level.

Univariate Normality Test				
Test Name	Variable	Statistic	p-value	Result
Shapiro-Wilk	Loss	0.8679	0.0729	✓
	GDD	0.9356	0.4700	✓
	GDP	0.9506	0.2163	✓
Cramer-von Mises	Loss	0.1006	0.0955	✓
	GDD	0.0501	0.4766	✓
	GDP	0.0622	0.3246	✓
Kolmogorov-Smirnov	Loss	0.2296	0.1078	✓
	GDD	0.1687	0.5145	✓
	GDP	0.1938	0.2956	✓
Anderson-Darling	Loss	0.6121	0.0824	✓
	GDD	0.3353	0.4383	✓
	GDP	0.3987	0.3026	✓
Multivariate Normality Test				
Test Name		Statistic	p-value	Result
Mardia	Skewness	6.8283	0.7415	✓
	Kurtosis	-1.0252	0.3053	
Henze-Zirkler	HZ statistics	0.5793	0.2698	✓
Royston	H statistics	5.0827	0.1643	✓

D Proof of Proposition 4.1

Since the loss random variable $\mathbf{L}_{(x,x+h]}$ can be expressed as:

$$\mathbf{L}_{(x,x+h]} = \begin{cases} 0, & X < x, \\ X - x, & x \le X < x + h, \\ h, & X \ge x + h, \end{cases} \tag{44}$$

we have

$$\begin{aligned} \text{RRL} &= \lim_{h\to 0} \frac{\mathsf{E}\big(\exp(\theta \mathbf{L}_{(x,x+h]})\mathbf{L}_{(x,x+h]}\big)}{\mathsf{E}\big(\exp(\theta \mathbf{L}_{(x,x+h]})\big)\mathsf{E}\big(\mathbf{L}_{(x,x+h]}\big)} \\ &= \lim_{h\to 0} \frac{\mathsf{E}\big(\exp\big(\theta(X-x)\big)\mathbb{I}_{\{x\le X<x+h\}}\big) + \mathsf{E}\big(\exp(h\theta)h\mathbb{I}_{\{X\ge x+h\}}\big)}{[\mathsf{E}\big(\mathbb{I}_{\{X<x\}}\big) + \mathsf{E}\big(\exp\big(\theta(X-x)\big)\mathbb{I}_{\{x\le X<x+h\}}\big) + \mathsf{E}\big(\exp(h\theta)\mathbb{I}_{\{X\ge x+h\}}\big)]\mathsf{E}\big(\mathbf{L}_{(x,x+h]}\big)}. \end{aligned}$$

By defining $E_u = \mathsf{E}\big(\exp\big(\theta(X-x)\big)\mathbb{I}_{\{x\le X<x+h\}}\big) + \mathsf{E}\big(\exp(h\theta)h\mathbb{I}_{\{X\ge x+h\}}\big)$, $E_{d_1} = \mathsf{E}\big(\mathbb{I}_{\{X<x\}}\big) + \mathsf{E}\big(\exp\big(\theta(X-x)\big)\mathbb{I}_{\{x\le X<x+h\}}\big) + \mathsf{E}\big(\exp(h\theta)\mathbb{I}_{\{X\ge x+h\}}\big)$ and $E_{d_2} = \mathsf{E}\big(\mathbf{L}_{(x,x+h]}\big)$, then

$$RR(x) = \lim_{h\to 0} \frac{E_u}{E_{d_1}E_{d_2}}.$$

Note that when $h \to 0$, $E_u \to 0$ and $E_{d_2} \to 0$, according to L'Hôpital's rule:

$$RR(x) = \lim_{h\to 0} \frac{\frac{d}{dx}E_u}{E_{d_1}\frac{d}{dx}E_{d_2} + E_{d_2}\frac{d}{dx}E_{d_1}}.$$

In addition,

$$\begin{aligned} E_{d_1}|_{h\to 0} &= 1, \\ \frac{d}{dx}E_{d_2}|_{h\to 0} &= 1 - F_X(x), \\ \frac{d}{dx}E_u|_{h\to 0} &= 1 - F_X(x), \\ E_{d_2}\frac{d}{dx}E_{d_1}|_{h\to 0} &= 0. \end{aligned}$$

Thus $RR(x) = \dfrac{1 - F_X(x)}{1 - F_X(x)} = 1$.

E Properties of MWPP

E.1 Positive risk loading and no ripoff

Proposition E.1. *$\Pi^w(X, \boldsymbol{Y}) \geq E(X)$ if and only if $\mathsf{Cov}\big(X, w(\boldsymbol{Y})\big) \geq 0$.*

Proof. It is straightforward that

$$\begin{aligned} \Pi^w(X, \boldsymbol{Y}) - \mathsf{E}(X) &= \frac{\mathsf{E}\big(w(\boldsymbol{Y})X\big)}{\mathsf{E}\big(w(\boldsymbol{Y})\big)} - \mathsf{E}(X) \\ &= \frac{\mathsf{E}\big(w(\boldsymbol{Y})X\big) - \mathsf{E}\big(w(\boldsymbol{Y})\big)\mathsf{E}(X)}{\mathsf{E}\big(w(\boldsymbol{Y})\big)} \\ &= \frac{\mathsf{Cov}\big(X, w(\boldsymbol{Y})\big)}{\mathsf{E}\big(w(\boldsymbol{Y})\big)}. \end{aligned} \tag{45}$$

Since $w(\boldsymbol{Y})$ is a positive function, we get

$$\Pi^w(X, \boldsymbol{Y}) \geq \mathsf{E}(X),$$

if and only if $\mathsf{Cov}\big(X, w(\boldsymbol{Y})\big) \geq 0$. □

In addition, note that

$$\frac{\mathsf{E}\big(w(\boldsymbol{Y})X\big)}{\mathsf{E}\big(w(\boldsymbol{Y})\big)} \leq \text{esssup}(X),$$

therefore, we have

$$\mathsf{E}(X) \leq \Pi^w(X, \boldsymbol{Y}) \leq \text{esssup}(X).$$

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