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A credibility-based Erlang mixture model for pricing crop reinsurance

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Abstract

Purpose – The purpose of this paper is to address some of the fundamental issues surrounding crop insurance ratemaking, from the perspective of the reinsurer, through the development of a scientific pricing framework.

Design/methodology/approach – The generating process of the historical loss cost ratio's (LCR's) are reviewed, and the Erlang mixture distribution is proposed. A modified credibility approach is developed based on the Erlang mixture distribution and the liability weighted LCR, and information from the observed data of the individual region/province is integrated with the collective experience of the entire crop reinsurance program in Canada.

Findings – A comprehensive data set representing the entire crop insurance sector in Canada is used to show that the Erlang mixture distribution captures the tails of the data more accurately compared to conventional distributions. Further, the heterogeneous credibility premium based on the liability weighted LCR's is more conservative, and provides a more scientific approach to enhance the reinsurance pricing.

Research limitations/implications – Credibility models are in the early stages of application in the area of agriculture insurance, therefore, the credibility models presented in this paper could be verified with data from other geographical regions.

Practical implications – The credibility-based Erlang mixture model proposed in this paper should be useful for crop insurers and reinsurers to enhance their ratemaking frameworks.

Originality/value – This is the first paper to introduce the Erlang mixture model in the context of agricultural risk modeling. Two modified versions of the Bühlmann-Straub credibility model are also presented based on the liability weighted LCR to enhance the reinsurance pricing framework.

Keywords Pricing, Crop insurance, Credibility theory, Erlang mixture distribution, Loss cost ratio, Ratemaking

Paper type Research paper

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1. Introduction

A sound and scientific agricultural (re)insurance pricing approach is essential for maintaining sustainable and viable risk management solutions for farmers, governments, insurers, and reinsurers alike. In the ratemaking process, the goal is to determine the fair risk charge, often through the use of available historical observations. However, agricultural insurance can be quite challenging to price due to unique challenges, including shortness of data, and highly variable losses from year to year, which are often geographically correlated across regions (Porth *et al.*, 2014). Pricing models that lack a scientific framework and possible consistency over time present a significant concern, including reinsurers who are currently faced with a competitive landscape, with lower premium rates for some business lines, and as such, tightening margins. Therefore, a robust pricing framework may help lead to steady underwriting performance and improve profitability over the long term.

The objective of this paper is to address some of the fundamental issues surrounding crop insurance ratemaking, from the perspective of the reinsurer, through the development of a scientific pricing framework that can be consistently and widely adopted by the agricultural sector. In doing so, this paper comprehensively examines the ratemaking process, including a review of the generating process of the historical loss cost ratio's (LCR's, which is defined as the ratio of indemnities to liabilities) in order to gain a better understanding of the underlying yield distributions. A parametric distribution known as the Erlang mixture is investigated to improve the goodness-of-fit. Further, the liability weighted LCR is introduced as a more conservative method to aggregate historical loss data. Based on these results, two modified versions of the Bühlmann-Straub credibility model are developed to enhance the pricing framework by combining information from the observed data of certain risk categories (i.e. historical LCR's for the individual region/province), also taking into consideration the experience from the collective risk pool (i.e. the entire crop insurance program in Canada across the nine other geographic regions/provinces).

1.1 Trending process

A fundamental question to be solved before ratemaking is a solid understanding of the generating process of the historical data. In time series analysis, scholars generally prefer to work with a stationary process, with constant mean and finite variance (Hamilton, 1994). However, in crop insurance, the yield and other related data series, such as the LCR, do not necessarily satisfy the assumption of stationarity. Instead, the crop time series data usually contains trends defined as either deterministic or stochastic, and the corresponding testing and detrending methods remain controversial. Many statistical tests have been proposed by econometricians over the past several decades, including the Augmented Dickey-Fuller (ADF) test (Dickey, 1976; Dickey and Fuller, 1979), the Phillips-Perron (PP) test (Perron, 1986; Phillips and Perron, 1988), the Dickey-Fuller with Generalized Least Squares (DF-GLS) Detrending test (Elliott *et al.*, 1996), and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski *et al.*, 1992).

In empirical study, researchers typically suggest applying linear trends (or higher order polynomial trends) to crop yield data with deterministic trends (Gallagher, 1986; Harri *et al.*, 2009; Luttrell and Gilbert, 1976; Sherrick *et al.*, 2004; Turvey and Zhao, 1999). Some other research has focussed on the presence of stochastic trends (Goodwin and Ker, 1998; Moss and Shonkwiler, 1993). A major limitation of these studies is that the current tools available for testing the trends in time series are based on an asymptotic assumption, which is not satisfied in the case of crop insurance due to

shortness of data (i.e. small sample size). In view of this limitation, the aforementioned four methods for testing trends in time series are reviewed in this study, particularly focussing on the implications of working with small sample sizes, as is the case in crop insurance. Through Monte Carlo simulation experiments, we show that in the small sample cases the size distortion is quite large and the power performance is poor for all four of the tests. This finding demonstrates the potential serious limitation in relying on these tests to obtain accurate and reliable information about trends when data samples are small. As such, a focus of this paper is on establishing a scientific approach to pricing that is robust under different trending processes.

1.2 Distribution frameworks

Deciding which distribution to use for modeling crop yield leads to another challenge. Past studies have tried to model yields with some known single parametric distributional attributes, including the normal distribution (Just and Weninger, 1999), log-normal distribution (Jung and Ramezani, 1999; Stocks, 2000; Tirupattur *et al.*, 1996), beta distribution (Nelson and Preckel, 1989; Sherrick *et al.*, 2004; Tirupattur *et al.*, 1996), gamma distribution (Gallagher, 1986), and Weibull distribution (Sherrick *et al.*, 2004). Nonparametric estimation methods have also been applied (Goodwin and Ker, 1998; Ker and Goodwin, 2000). As consensus has yet to be reached regarding yield modeling, this area of research remains of central importance given the fundamental importance it serves in the ratemaking process (Sherrick *et al.*, 2004).

One caution in distribution fitting is working with highly parameterized models, which can lead to overfittings, and hence poor forecasting and inadequate pricing. Mixture models studied by researchers such as Lanoue *et al.* (2010), Woodard and Sherrick (2011b), and Yang (2011) have indicated that in comparison to the single distribution models, these models offer greater flexibility and have better out-of-sample performances. In this paper we explore another mixture model, known as the Erlang mixture, for application in agricultural risk modeling. We also investigate the associated desirable properties.

To test the proposed Erlang mixture distribution, different models are compared in terms of likelihoods and BIC values using a comprehensive data set that represents the entire crop insurance sector of Canada. The data includes historical indemnities and liabilities (from which the LCR's are calculated), over 1974-2009, across 276 crop types, and ten geographic regions (provinces).

1.3 Modified credibility approach

Credibility theory has received little attention in the area of agricultural insurance pricing, with the exception of two papers, Josephson *et al.* (2000), and Pai *et al.* (2014). In this paper, a modified Bühlmann-Straub Credibility Model is developed based on the Erlang mixture distribution, in an effort to enhance the reinsurance pricing framework. The motivation of utilizing a credibility approach is to help address the challenge of shortness of data, and improve the statistical estimates of the expected losses. The credibility approach combines information from the historical LCR's for the individual region/province, as well as the collective experience for the entire crop reinsurance program in Canada across nine other geographic regions/provinces. An empirical pricing example is presented to support the proposed pricing framework.

The remainder of this paper is as follows. In the next section, four statistical tests to examine the trending processes of crop loss data are considered, and the corresponding simulation results are provided. In Section 3, a statistical description of the data set is

reviewed, and the proposed Erlang mixture distribution family is presented in Section 4. Based on these results, the modified Bühlmann-Straub Credibility approach is developed in Section 5, and the liability weighted LCR is proposed as a more conservative way to aggregate historical losses for the pricing model. Section 6 provides some empirical evidence regarding the appropriateness of the credibility models for pricing crop insurance/reinsurance policies. Finally, the study is concluded in Section 7 with some empirical recommendations and future research directions.

2. Trend testing

2.1 Deterministic trends and stochastic trends

For a time series y_t and $t \geqslant 0$, a model with a deterministic trend is defined as:

$$y_t = d_t + u_t \tag{1}$$

where d_t is some deterministic function of t, and u_t is some stationary process. This model can also be described as a trend-stationary process or an integrated of order zero process ($I(0)$ process). On the other hand, if u_t contains an autoregressive unit root, a model with a stochastic trend is defined as:

$$u_t = \sum_{i=0}^{p} \phi_i u_{t-i} + v_t \tag{2}$$

where v_t is some stationary process, and $\text{Max}(\phi_i, i = 1,2, \ldots, p) = 1$. This model can also be defined as a difference-stationary process or an integrated of order one process ($I(1)$ process).

Numerous tests have been proposed by econometricians for testing unit roots. They are known as unit root tests, with the null hypothesis that the time series is $I(1)$ against the alternative that the time series is $I(0)$. Or conversely, if the null hypothesis is that the time series is $I(0)$ against the alternative hypothesis that the series is $I(1)$, they are known as stationary tests. Under a Gaussian process assumption, Elliott *et al.* (1996) proved that asymptotically many tests manage to achieve power functions that are extremely close to the power envelope given by Neyman-Pearson Lemma, which is the upper bound for the power function of any tests based on the same likelihood. The asymptotic assumption is highly questionable for most real world systems. Monte Carlo simulation is used in many studies to examine the finite sample performance of these tests (DeJong *et al.*, 1991; Phillips and Perron, 1988; Schwert, 1989). However, these studies are limited in the sense that they all consider sample sizes larger than 100, which is still relatively large compared to the particular situation in crop insurance where data of only several decades (with annual observations) are available at best. Hence, a better understanding of how these tests perform in sample sizes far smaller than the asymptotic cases becomes critically important, prior to executing these tests for the purpose of testing trends.

2.2 Simulation results for tests

This subsection reviews the four most frequently used tests for unit roots, followed by a simulation study.

2.2.1 Unit root tests. ADF test: Dickey (1976), Dickey and Fuller (1979) developed the ADF test based on the $AR(p)$ model defined as:

$$y_t = \beta' D_t + \phi y_{t-1} + \sum_{j=1}^{p} \psi_j \Delta y_{t-j} + \varepsilon_t \tag{3}$$

with $H_0 : \phi = 1$ against $H_1 : |\phi| < 1$, where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

PP test: Perron (1986), Phillips and Perron (1988) proposed a unit root test based on a nonparametric regression model defined as:

$$\Delta y_t = \beta' D_t + \phi y_{t-1} + u_t \tag{4}$$

where u_t is $I(0)$, and allows for heteroskedasticity and correlation.

DF-GLS test: Elliott *et al.* (1996) proposed a family of tests whose asymptotic power functions are tangent to the Gaussian power envelop by considering the asymptotic approximation based on the local-to-unity alternative $c = T(\phi - 1)$. Among this family of tests, the DF-GLS test, which is a modified and efficient version of the ADF t-test, is shown to improve power given the same sample size, and when the trending process of the time series is unknown.

2.2.2 Stationary test. KPSS test: Kwiatkowski *et al.* (1992) proposed a test with null hypothesis that the time series is trend stationary, against an alternative that it has a unit root. The model is specified as:

$$y_t = \beta' D_t + \mu_t + u_t \tag{5}$$

$$\mu_t = \mu_{t-1} + \varepsilon_t \tag{6}$$

$$\varepsilon_t \sim WN(0, \sigma_{\varepsilon_t}^2) \tag{7}$$

with $H_0 : \sigma_\varepsilon^2 = 0$ against $H_1 : \sigma_\varepsilon^2 > 0$, and u_t is a stationary process, which is allowed to be heteroskedastic.

To study the size and power performance of these four tests, we conduct simulation experiments based on the following model with 1,000 replicates:

$$y_t = \phi y_{t-1} + u_t; \qquad u_t \sim N(0, \sigma^2) \tag{8}$$

The size distortions and power performances of these tests are listed in Tables I-IV, where the constant model refers to $D_t = 1$, and the trend model refers to $D_t = (1, t)'$. Each row corresponds to various sample sizes (e.g. 25, 50, etc. up to 1,000), and the columns are the number of lags chosen in the testing model. The results are compared to a nominal size of 0.05. These tables show that when the sample size is larger than 500, all four tests perform well as indicated by a small size distortion and a high power close to one, even if the D_t is not correctly specified in the test. However, when the sample size is smaller than 50, the test results have larger size distortions and unsatisfying power performances, namely, they provide inaccurate information regarding the trends of the processes.

Moreover, the size distortions and power performances are much more sensitive to the misspecification of the trend when the sample sizes are smaller than 250, which could result in misleading testing results. For example, in Table I, when the sample size is 50, the average power of the ADF test is 158.24 percent higher when the trend model is correctly specified. Additionally, for the KPSS stationary test, Monte Carlo simulation results are found to be consistent with the findings reported by Kwiatkowski *et al.* (1992) for a sample size equal to 30. Therefore, we conclude that the current statistical tests for trending processes are not revealing when sample sizes are prohibitively small, as is the challenge faced in crop insurance.

Sample/lags	2	4	6	8	10
Size distortion (ADF): constant model					
$T = 25$	0.0568	0.0715	0.0985	0.1497	0.2642
$T = 50$	0.0519	0.0533	0.0556	0.0629	0.0761
$T = 100$	0.0505	0.0513	0.0515	0.0512	0.0547
$T = 250$	0.0482	0.0461	0.0500	0.0495	0.0458
$T = 500$	0.0487	0.0529	0.0458	0.0542	0.0508
$T = 1{,}000$	0.0439	0.0493	0.0454	0.0426	0.0485
Size distortion (ADF): trend model					
$T = 25$	0.0806	0.1149	0.1704	0.3036	0.5553
$T = 50$	0.0625	0.0685	0.0802	0.0910	0.1148
$T = 100$	0.0554	0.0580	0.0609	0.0653	0.0640
$T = 250$	0.0472	0.0553	0.0544	0.0506	0.054
$T = 500$	0.0493	0.0524	0.0509	0.0523	0.0484
$T = 1{,}000$	0.0490	0.0528	0.0531	0.0542	0.0492
Power performance (ADF): constant model					
$T = 25$	0.2164	0.2484	0.3118	0.4319	0.6041
$T = 50$	0.2583	0.2475	0.2585	0.2674	0.2865
$T = 100$	0.4440	0.386	0.3594	0.3321	0.3128
$T = 250$	0.9518	0.9095	0.8684	0.8195	0.7663
$T = 500$	1	0.9999	0.9998	0.9992	0.9978
$T = 1{,}000$	1	1	1	1	1
Power performance (ADF): trend model					
$T = 25$	0.0836	0.1138	0.1793	0.3096	0.3088
$T = 50$	0.0908	0.0916	0.0882	0.1024	0.1069
$T = 100$	0.1674	0.1478	0.1358	0.1274	0.1255
$T = 250$	0.7499	0.6568	0.5750	0.4941	0.4911
$T = 500$	0.9998	0.9971	0.9930	0.9805	0.9813
$T = 1{,}000$	1	1	1	1	1

Notes: The nominal size is 0.05, and the alternative hypothesis is $\phi = 0.9$. "Lags" refers to the order of integration in the ADF regression model (value of p in Equation (3))

Table I.
Size distortions and power performances of ADF test

3. Data description and its properties

By using a unique and a comprehensive data set that covers the entire crop insurance sector of Canada (provided by Agriculture and Agri-Food Canada), this section provides an in-depth analysis on the statistical characteristics of the historical LCR's (Section 3.1) and the trend testing results (Subsection 3.2). The data set includes actual indemnities and liabilities from 1974 through 2009, across 276 crop types, and ten geographic regions (i.e. provinces) in Canada. The ten provinces considered include Alberta (AB), Manitoba (MB), Ontario (ON), British Columbia (BC), New Brunswick (NB), Nova Scotia (NS), Prince Edward Island (PEI), Quebec (QC), Saskatchewan (SK), and Newfoundland and Labrador (NFLD).

In this paper, the indemnities and liabilities are aggregated to a provincial level. From this, the LCR's are calculated as the ratio of indemnities to liabilities. Reinsurers commonly utilize the LCR to normalize the loss exposure of the underlying risk profile, and to compute premiums. This normalization is important because historically there have been significant increases in liabilities and yields, increasing program participation, as well as improvements in biotechnology, farming practices, etc. (Coble *et al.*, 2008; Harri *et al.*, 2009; Miranda and Glauber, 1997; Sherrick *et al.*, 2004; Woodard *et al.*, 2012; Woodard and Sherrick, 2011a) Given that reinsurers often face

Sample/lags	Z_α		Z_τ	
	l_4	l_{12}	l_4	l_{12}
Size distortion (PP): constant model				
$T = 25$	0.0474	0.0447	0.0608	0.0587
$T = 50$	0.0574	0.0533	0.0593	0.0588
$T = 100$	0.0510	0.0568	0.0526	0.0589
$T = 250$	0.0513	0.0576	0.0535	0.0528
$T = 500$	0.0529	0.0562	0.0517	0.0577
$T = 1{,}000$	0.0508	0.0507	0.052	0.0517
Size distortion (PP): trend model				
$T = 25$	0.0365	0.0356	0.0607	0.0578
$T = 50$	0.0587	0.0540	0.0644	0.0626
$T = 100$	0.0659	0.0618	0.0640	0.0619
$T = 250$	0.0618	0.0627	0.0625	0.0585
$T = 500$	0.0539	0.0584	0.0570	0.0589
$T = 1{,}000$	0.056	0.0534	0.0576	0.0545
Power performance (PP): constant model				
$T = 25$	0.0883	0.0945	0.0839	0.0866
$T = 50$	0.1925	0.1892	0.1479	0.1441
$T = 100$	0.4759	0.4845	0.3591	0.3692
$T = 250$	0.9924	0.9912	0.9711	0.9701
$T = 500$	1.0000	1.0000	1.0000	1.0000
$T = 1{,}000$	1.0000	1.0000	1.0000	1.0000
Power performance (PP): trend model				
$T = 25$	0.0467	0.0479	0.0722	0.0713
$T = 50$	0.1003	0.0959	0.0999	0.0961
$T = 100$	0.2769	0.2693	0.2350	0.2329
$T = 250$	0.9098	0.9053	0.8644	0.8588
$T = 500$	1.0000	1.0000	1.0000	1.0000
$T = 1{,}000$	1.0000	1.0000	1.0000	1.0000

Table II.
Size distortions and power performances of PP test

Notes: The nominal size is 0.05, and the alternative hypothesis is $\phi = 0.9$. The test statistics Z_α and Z_τ are calculated according to the expressions in Dickey and Fuller (1979). $l_4 = \text{floor}[4(T/100)^{1/4}]$ and $l_{12} = \text{floor}[12(T/100)^{1/4}]$

constraints regarding the detail of the crop data provided to them, loss modeling becomes difficult. Therefore, a pricing model that is robust under different trending processes is imperative.

3.1 Statistical characteristics of the LCR data

This subsection describes the statistical characteristics of the Canadian crop LCR data set used in this paper. For all ten provinces, the data is positively skewed, and most provinces have high kurtosis, reflecting a heavy tail property. Empirical value at risks (VaR's) and conditional tail expectations (CTE's) at different levels also report heavy tails. In crop insurance, a LCR > 0.25 is usually suggestive of a substantial and widespread loss. As shown in Table V, the tail quantiles for most provinces are higher than 0.25, which suggests the catastrophic nature of the loss experience. This is particularly true for the three largest crop insurance provinces, Alberta, Manitoba, and Saskatchewan, which together comprise more than 50 percent of the crop insurance program in Canada. These statistical characteristics are important to consider in developing a reinsurance pricing model, where it is important to ensure that peak loss experiences are accounted for in loading and reserving considerations.

Sample/lags	2	4	6	8	10
Size distortion (DF-GLS): constant model					
$T = 25$	0.1276	0.0983	0.0881	0.0841	0.1143
$T = 50$	0.0914	0.0872	0.0782	0.0693	0.0633
$T = 100$	0.0670	0.0705	0.0633	0.0596	0.0597
$T = 250$	0.0612	0.0600	0.0583	0.0554	0.0560
$T = 500$	0.0539	0.0554	0.0545	0.0485	0.0557
$T = 1{,}000$	0.0535	0.0534	0.054	0.0489	0.0515
Size distortion (DF-GLS): trend model					
$T = 25$	0.0594	0.0353	0.0243	0.0232	0.0523
$T = 50$	0.0395	0.0324	0.0252	0.0202	0.0170
$T = 100$	0.0479	0.0415	0.0376	0.0450	0.0420
$T = 250$	0.0508	0.0494	0.0496	0.0503	0.0445
$T = 500$	0.0491	0.0486	0.0490	0.0452	0.0438
$T = 1{,}000$	0.0410	0.0460	0.0475	0.0446	0.0511
Power performance (DF-GLS): constant model					
$T = 25$	0.2180	0.1613	0.1156	0.1057	0.1257
$T = 50$	0.3010	0.2352	0.1881	0.1516	0.1216
$T = 100$	0.5326	0.4442	0.3805	0.314	0.2717
$T = 250$	0.8852	0.8427	0.7800	0.7301	0.6768
$T = 500$	0.9806	0.9642	0.9451	0.9132	0.8957
$T = 1{,}000$	0.9991	0.9975	0.9934	0.9879	0.9790
Power performance (DF-GLS): trend model					
$T = 25$	0.0676	0.0384	0.026	0.0224	0.0492
$T = 50$	0.0724	0.0511	0.0355	0.0244	0.0174
$T = 100$	0.1907	0.1571	0.1267	0.1378	0.1041
$T = 250$	0.8207	0.7345	0.6478	0.5604	0.4837
$T = 500$	0.9910	0.9845	0.9677	0.9479	0.9197
$T = 1{,}000$	1.0000	0.9999	0.9995	0.9990	0.9970

Notes: The nominal size is 0.05, and the alternative hypothesis is $\phi = 0.9$. "Lags" refers to the order of integration in the ADF regression model (value of p in Equation (3))

Table III.
Size distortions and power performances of DF-GLS test

3.2 Trend testing results

As discussed in the previous sections, the first step of loss modeling is to understand the data trending process, and then eliminate the trends accordingly. Various detrending methods can result in substantial differences in the underlying data series, and at times lead to misspecified estimators of the distributions (Zapata and Rambaldi, 1989). Therefore, different trends require correspondingly appropriate detrending methods. When the trend is deterministic, the data should be detrended by eliminating the time trend, yet, when the trend is stochastic, first differences should be used.

In this paper, we begin the pricing process using the DF-GLS unit root test, followed by the KPSS stationary test. The DF-GLS test is shown to have the best performance for small sample cases among the three unit root tests according to Tables I and II. Table VI shows the test results of the LCR data for each province according to the DF-GLS test and KPSS test. Five of the ten provinces are found to have inconsistent results for the two tests. This makes it difficult to draw conclusions from the tests in order to determine the most suitable detrending method. As such, a model that is robust under various detrending methods can be very helpful to ensure a sound pricing framework, and avoid misclassification of the trending process.

Sample/lags	l_0	l_4	l_1
Size distortion (KPSS): constant model			
$T = 25$	0.0518	0.0311	0.0369
$T = 50$	0.0497	0.0402	0.0412
$T = 100$	0.0502	0.0441	0.0457
$T = 250$	0.0494	0.0501	0.0503
$T = 500$	0.0482	0.0460	0.0495
$T = 1{,}000$	0.0511	0.0461	0.0473
Size distortion (KPSS): trend model			
$T = 25$	0.0574	0.0399	0.0387
$T = 50$	0.0535	0.0431	0.0406
$T = 100$	0.0531	0.0453	0.0464
$T = 250$	0.0571	0.0467	0.0500
$T = 500$	0.0522	0.049	0.0505
$T = 1{,}000$	0.0514	0.0515	0.0506
Power performance (KPSS): constant model			
$T = 25$	0.1191	0.0877	0.0842
$T = 50$	0.3080	0.2560	0.2576
$T = 100$	0.6048	0.5281	0.5250
$T = 250$	0.9117	0.8498	0.8403
$T = 500$	0.9888	0.9657	0.9630
$T = 1{,}000$	0.9998	0.9944	0.9949
Power performance (KPSS): trend model			
$T = 25$	0.0742	0.0512	0.0498
$T = 50$	0.1406	0.0993	0.1016
$T = 100$	0.3859	0.2988	0.2897
$T = 250$	0.8451	0.7558	0.7568
$T = 500$	0.9872	0.9628	0.9634
$T = 1{,}000$	0.9999	0.9968	0.9965

Table IV.
Size distortions and power performances of KPSS test

Notes: The nominal size is 0.05, and the alternative hypothesis is $\phi = 0.9$. $l_4 = \text{floor}[4(T/100)^{1/4}]$ and $l_{12} = \text{floor}[12(T/100)^{1/4}]$

LCR	AB	MB	ON	BC	NB	NS	PEI	QC	SK	NFLD
μ	0.13	0.09	0.07	0.08	0.14	0.05	0.08	0.06	0.11	0.14
σ	0.09	0.07	0.05	0.05	0.11	0.03	0.05	0.04	0.09	0.09
γ_1	2.31	1.77	1.39	0.58	1.79	0.65	1.18	1.25	1.97	0.66
γ_2	9.33	5.16	4.02	2.36	7.39	3.11	4.96	4.84	6.93	2.76
$VaR_{0.85}$	0.20	0.20	0.11	0.14	0.21	0.08	0.14	0.10	0.19	0.25
$VaR_{0.9}$	0.23	0.23	0.16	0.16	0.30	0.09	0.15	0.12	0.22	0.28
$VaR_{0.95}$	0.33	0.29	0.19	0.18	0.37	0.12	0.18	0.15	0.40	0.34
$CTE_{0.85}$	0.29	0.26	0.17	0.17	0.34	0.11	0.17	0.13	0.29	0.31
$CTE_{0.9}$	0.35	0.28	0.19	0.18	0.40	0.12	0.19	0.15	0.36	0.34
$CTE_{0.95}$	0.48	0.32	0.19	0.18	0.57	0.13	0.25	0.19	0.43	0.35

Table V.
Statistical descriptions of the LCR data

Notes: μ, Mean; σ, standard deviation; γ_1, skewness; γ_2, kurtosis

4. Erlang mixture distribution

In this section, we propose using the Erlang mixture distribution for modeling agricultural crop data. We will also provide evidence that this distribution is able to capture the tails of the data more accurately. To the best of our knowledge this

distribution has not been studied in agricultural risk modeling and (re)insurance pricing. The Erlang mixture family is a very important class of distribution because theoretically it is dense in the space of positive distributions. In other words, there always exists a series of mixture Erlangs that converges in distribution to an arbitrary positive distribution (Tijms, 1994).

The probability distribution function (p.d.f.) of the mixed Erlang model is defined as:

$$f(x|\theta,\alpha)=\sum_{i=1}^{M}\alpha_i\frac{x^{r_i-1}e^{-x/\theta}}{\theta^{r_i}(r_i-1)!}\quad x>0$$

where the shape parameters $r_i'S,(i=1,\ldots,M)$ are increasing integers. The number of mixing Erlang distributions M, the mixing coefficients $\alpha_i'S$, $(i=1,\ldots,M)$, and the scale parameter θ, are found by the Expectation-Maximization Algorithm proposed by Lee and Lin (2010) to maximize the log-likelihood functions.

To test the plausibility of this distribution for modelling crop data, we resort to the commonly used criterion known as the Bayesian information criterion (BIC). BIC was first introduced by Schwarz (1978), and is defined as:

$$\text{BIC}=-2\log(\hat{\mathcal{L}})+K\cdot\log(N)$$

where $\hat{\mathcal{L}}$ is the estimated value of the likelihood function, K is the number of parameters in the model, and N is the sample size. The advantage of using BIC as the model selection criterion is that it is valid not only for nested models, but, also for non-nested models (Burnham and Anderson, 2002).

In order to compare the goodness-of-fit of the Erlang mixture distribution model, some commonly used distributions, such as gamma, Weibull, beta, normal and loglogistic are also considered as benchmarks. Further, the Erlang mixture model is considered with respect to the two different detrending processes considered previously. The first method involves regressing the LCR's with respect to time (hereafter referred to as regressed data), and the second method involves differencing the LCR's (hereafter referred to as differenced data). The fitting results for the regressed data as well as the differenced data are found in Table VII, respectively. For both detrending methods, the Erlang mixture distributions has the highest likelihoods and the lowest BIC's for all the ten provinces. While the Erlang mixture distributions is superior, the gamma and Weibull distributions also perform well.

As mentioned earlier, reinsurers are faced with the challenge of asymmetric information. Reinsurers often only have access to limited historical time series and often this data are highly aggregated with mixed coverage levels and rates through time, which may mask potential trends. From this perspective, within the framework of a parametric approach, the Erlang distribution has a promising advantage in that it

LCR	AB	MB	ON	BC	NB	NS	PEI	QC	SK	NFLD
Time trend	1	1	1	1	1	1	1	1	1	1
DF-GLS	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
KPSS	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$

Notes: Constant model, significant level = 0.05. 1 in the first line refers to an insignificant time trend, $I(0)$ refers to a deterministic trend, and $I(1)$ is a stochastic trending process

Table VI. Test results of the LCR data

Province	Criteria	Erlang	Gamma	Weibull	Beta	Loglogistic	Normal
Maximum likelihood estimations: regressed data							
AB	Likelihood	135.57	50.16	49.76	3.06	43.95	37.92
	BIC	−242.48	−93.16	−92.36	1.05	−80.74	−68.67
MB	Likelihood	200.84	57.61	57.12	4.26	52.17	42.85
	BIC	−383.77	−108.06	−107.07	−1.35	−97.16	−78.53
ON	Likelihood	216.22	76.93	75.99	10.62	70.57	59.01
	BIC	−414.53	−146.70	−144.80	−14.08	−133.97	−110.85
BC	Likelihood	334.73	50.88	52.74	−4.11	43.68	59.95
	BIC	−651.53	−94.59	−98.32	15.38	−80.18	−112.74
NB	Likelihood	205.84	42.58	41.36	3.00	34.58	29.19
	BIC	−397.34	−77.99	−75.56	1.16	−62.00	−51.21
NS	Likelihood	276.44	75.68	75.72	1.69	67.53	75.76
	BIC	−524.22	−144.18	−144.28	3.78	−127.90	−144.36
PEI	Likelihood	175.53	60.69	60.75	1.52	52.81	58.94
	BIC	−333.14	−114.22	−114.34	4.12	−98.45	−110.71
QC	Likelihood	255.88	70.68	71.08	1.85	63.71	69.00
	BIC	−493.84	−134.19	−134.99	3.48	−120.25	−130.84
SK	Likelihood	151.23	50.47	49.73	4.19	43.95	35.20
	BIC	−284.54	−93.77	−92.29	−1.22	−80.74	−63.23
NFLD	Likelihood	265.15	35.67	34.53	−2.60	25.24	35.21
	BIC	−512.38	−64.18	−61.90	12.37	−43.31	−63.25
Maximum likelihood estimations: differenced data							
AB	Likelihood	428.14	−2.65	2.30	−15.99	−9.75	29.96
	BIC	−842.07	12.42	2.50	39.08	26.60	−52.81
MB	Likelihood	251.84	14.07	17.10	−12.15	6.35	33.47
	BIC	−485.90	−21.04	−27.09	31.41	−5.60	−59.84
ON	Likelihood	318.79	35.95	36.48	−2.71	26.24	46.59
	BIC	−626.90	−64.80	−65.84	12.53	−45.37	−86.07
BC	Likelihood	332.68	44.39	45.38	−4.06	34.88	57.41
	BIC	−651.13	−81.66	−83.64	15.23	−62.66	−107.71
NB	Likelihood	384.53	−10.61	−5.81	−13.55	−17.92	22.17
	BIC	−754.83	28.33	18.74	34.22	42.96	−37.23
NS	Likelihood	347.04	40.07	43.08	−3.18	32.65	57.95
	BIC	−679.86	−73.02	−79.05	13.47	−58.19	−108.80
PEI	Likelihood	372.08	17.06	21.23	−10.42	10.02	41.98
	BIC	−729.93	−27.01	−35.35	27.96	−12.94	−76.85
QC	Likelihood	398.93	32.67	35.54	−7.79	25.10	50.08
	BIC	−776.53	−58.23	−63.97	22.69	−43.08	−93.05
SK	Likelihood	349.67	7.06	10.24	−15.26	−0.41	26.59
	BIC	−685.11	−7.01	−13.36	37.63	7.92	−46.07
NFLD	Likelihood	353.31	13.05	15.30	−12.66	5.03	28.42
	BIC	−685.28	−18.99	−23.49	32.43	−2.95	−49.72

Table VII.
Maximum likelihood estimating results for LCR data under two different detrending methods

is robust regardless of the detrending method, which is not the case with the other distributions. Therefore, the Erlang mixture distribution may help to improve the ratemaking and loss reserving process for insurers and reinsurers, particularly when faced with small data samples as in crop insurance.

5. Modified Bühlmann-Straub credibility models

The objective of this section is to present an enhanced pricing methodology for crop (re)insurance through the integration of a modified credibility model, and the Erlang mixture model. The credibility model is a widely accepted ratemaking tool used by

actuaries in the property and casualty sectors to price insurance policies for risk exposures such as automobile and health.

The motivation of using a credibility approach is to enhance pricing through the addition of supplemental information. For example, suppose we are interested in predicting future claims of a particular risk class, the traditional pure risk premium approach involves using past claim experience data pertinent to the risk class. However, in addition to using the history of claims for the given risk class, credibility theory (see Bühlmann, 1967; Bühlmann and Straub, 1970) argues that the prediction power can be improved by also exploiting the claim experience of the collective pool of risk classes with similar characteristics. By restricting the class of estimators to be a linear combination of past observations, an optimal credibility-based prediction estimator that minimizes the squared difference between the predicted values and the individual premium can be obtained. This results in an estimator that is simple and intuitive. The credibility estimator becomes a linear combination between the individual risk class claim experience and the collective claim experience with relative weights dependent on the credibility of the claim experience of the risk class and the collective risk pool. In the special case in which the experience of a risk class is fully credible, then the experience of the risk class is solely used for future loss prediction.

The classical Bühlmann-Straub credibility model assumes that claims are independent conditional on a given risk category, and the random variables characterizing the risk profiles are identically and independently distributed (see Bühlmann and Gisler, 2005). However, in the application of agricultural insurance, the identical conditional distribution assumption may not be appropriate because there are some priori differentiations for each province. This priori differentiation intuitively makes sense since each province is faced with different weather, technical, and economic situations, therefore, their risk characteristics should contain some priori differences. In addition to giving a priori differentiation, we obtain the credibility premium based on two different risk assumptions for the loss data, including a homogeneous risk assumption and a heterogeneous risk assumption. The two modified Bühlmann-Straub credibility models are discussed next.

For $i = 1,2, \ldots, d$ and $j = 1,2, \ldots, T$, where d and T denote, respectively, the number of risk categories (i.e. number of provinces in our context), and the number of years of data, we define the following notation:

- X_{ij}: LCR of risk i in year j;
- I_{ij}: indemnity of risk i in year j;
- L_{ij}: liability (risk exposure) of risk i in year j; and
- Θ_i: the risk profile parameter of the ith risk.

In our context, the LCR $X_{ij} = I_{ij}/L_{ij}$ is the basic random variable for the credibility rating. The parameter Θ_i is defined as a random variable that describes the risk characteristics for the ith risk. The assumptions underlying the homogeneous and heterogeneous risk models, and the resulting credibility premiums are stated below.

M1: Homogeneous risk model:

A1: $\mathbf{X} = \{X_{ij} : i = 1,2,\ldots,d;\ j = 1,2,\ldots,T\}$ are independent conditional on Θ_i, with its (conditional) mean and (conditional) variance given, respectively, by:

$$E[X_{ij}|\Theta_i] = a_i \mu(\Theta_i) \tag{9}$$

$$Var[X_{ij}|\Theta_i] = a_i \frac{\sigma_1^2(\Theta_i)}{b_i} \tag{10}$$

The parameters a_i and b_i are known constants describing the priori differentiation between each risk (or province) i and $\mu(\Theta_i)$, and $\sigma_1^2(\Theta_i)$ are functions of Θ_i.

A2: $(\Theta_1,\mathbf{X_1})$, $(\Theta_2,\mathbf{X_2})\ldots,(\Theta_d,\mathbf{X_d})$ are independent, and $\Theta_1,\Theta_2,\ldots,\Theta_d$ are independent and identically distributed.

As shown in Appendix 2, for the two assumptions stated above, the credibility premium for the ith risk class (province) is given by:

$$\mathrm{CreP}_i^{\mathrm{Hom}} = Z_i^{\mathrm{Hom}} \bar{X}_i^{\mathrm{Hom}} + (1 - Z_i^{\mathrm{Hom}}).\mu_i \tag{11}$$

In the above formula:

- $\bar{X}_i^{\mathrm{Hom}}$, which denotes the historical average of the LCR of risk category i, is given by:

$$\bar{X}_i^{\mathrm{Hom}} = \frac{1}{T}\sum_{j=1}^{T} X_{ij} \tag{12}$$

- $\mu_0 = E(\mu(\Theta_i))$ can be interpreted as the risk premium for the entire collective risk pool (without taking into consideration the priori difference between the provinces).
- Z_i^{Hom} is the credibility factor, and captures the weight that is assigned to the historical data of the ith province in the credibility premium. Formally this is defined as:

$$Z_i^{\mathrm{Hom}} = \frac{a_i b_i \cdot T}{a_i b_i \cdot T + \kappa^{\mathrm{Hom}}} \tag{13}$$

with the credibility coefficient k^{Hom} given by:

$$\kappa^{\mathrm{Hom}} = \frac{v^{\mathrm{Hom}}}{a^{\mathrm{Hom}}}, \qquad \text{where } v^{\mathrm{Hom}} = E[\sigma_1^2(\Theta_i)], \quad a^{\mathrm{Hom}} = Var[\mu(\Theta_i)] \tag{14}$$

A more intuitive expression for the credibility factor is to use the following equivalent representation:

$$Z_i^{\mathrm{Hom}} = \frac{T}{T + \tilde{\kappa}^{\mathrm{Hom}}} \qquad \text{where } \tilde{\kappa}_i^{\mathrm{Hom}} = \frac{E[Var[X_{ij}|\Theta_i]]}{a_i b_i Var(E[X_{ij}|\Theta_i])}$$

This demonstrates that the credibility factor depends explicitly on three factors. More specifically, the credibility factor increases as:

- the number of observations T increases;
- the variability within the risk classes (as measured by $E[Var[X_{ij}|\Theta i]]$) decreases; and
- the heterogeneity of the collective risk pool (as measured by $Var(E[X_{ij}|\Theta_i])$) increases.

The above interpretations are intuitive. The credibility premium (11) is a weighted average between the observed LCR average of the ith risk category and the risk premium for the entire collective risk pool. The past observations become more credible and lead to a higher credibility factor with more observations, a smaller within risk category variability, or a larger between risk category variability.

M2: Heterogeneous risk model:

A1: $\mathbf{X} = \{X_{ij} : i = 1,2,\ldots,d;\ j = 1,2,\ldots,T\}$ are independent conditional on $\boldsymbol{\Theta}_i$, with its (conditional) mean and (conditional) variance given, respectively, by:

$$E[X_{ij}|\boldsymbol{\Theta}_i] = a_i \mu(\boldsymbol{\Theta}_i) \tag{15}$$

$$Var[X_{ij}|\boldsymbol{\Theta}_i] = a_i \frac{\sigma_2^2(\boldsymbol{\Theta}_i)}{b_i \cdot L_{ij}} \tag{16}$$

where a_i and b_i are interpreted similarly as in the previous model and $\sigma_2^2(\boldsymbol{\Theta}_i)$ is another function in term of $\boldsymbol{\Theta}_i$.

A2: $(\boldsymbol{\Theta}_1,\mathbf{X_1}),(\boldsymbol{\Theta}_2,\mathbf{X_2})\ldots,(\boldsymbol{\Theta}_T,\mathbf{X_d})$ are independent, and $\boldsymbol{\Theta}_1,\boldsymbol{\Theta}_2,\ldots,\boldsymbol{\Theta}_d$ are independent and identically distributed.

To capture the heterogeneity of the data, the conditional variance of X_{ij} depends on parameters a_i, b_i, a function in terms of $\boldsymbol{\Theta}_i$ as in the homogeneous risk model, and also on an extra factor L_{ij}, which measures the year j risk exposure of the ith province. Under the above assumptions, Appendix 2 similarly establishes that the credibility premium for province i becomes:

$$\text{CreP}_i^{\text{Het}} = Z_i^{\text{Het}} \bar{X}_i^{\text{Het}} + (1 - Z_i^{\text{Het}})\mu_i \tag{17}$$

where:

$$\bar{X}_i^{\text{Het}} = \sum_{j=1}^{T} \frac{L_{ij}}{L_{i\cdot}} X_{ij} \tag{18}$$

$$Z_i^{\text{Het}} = \frac{a_i b_i L_{i\cdot}}{a_i b_i L_{i\cdot} + \kappa^{\text{Het}}} \tag{19}$$

$$L_{i\cdot} = \sum_{j=1}^{T} L_{ij}, \quad \mu_i = a_i \mu_0 \tag{20}$$

$$\kappa^{\text{Het}} = \frac{v^{\text{Het}}}{a^{\text{Het}}}, \quad v^{\text{Het}} = E[\sigma_2^2(\boldsymbol{\Theta}_i)], \quad a^{\text{Het}} = Var[\mu(\boldsymbol{\Theta}_i)] \tag{21}$$

The interpretations of these parameters are similar to the previous model. The main difference between the homogeneous and heterogeneous assumptions is that consideration is given to the improvement of the risk exposure in each year in the heterogeneous model. This is important because trends in the risk exposure, as a result of increasing yields, increasing commodity prices, etc. leads to considerable variability. This makes the heterogeneous credibility model more reasonable by factoring both geographic and time variations of liability into the premium calculation.

We also note that if $a_i - b_i = 1$, the homogeneous risk model becomes the Bühlmann model (see Bühlmann, 1967), and the heterogeneous risk model becomes the Bühlmann-Straub model (see Bühlmann and Straub, 1970). In these traditional credibility models, the credibility factor Z is the same for all risk categories. However, in the modified models introduced in this paper, different credibility factors are given to each risk class due to the priori differentiation characterized by parameters a_i and b_i. Therefore, we expect the modified models to provide better credibility adjusted future premiums based on past experiences, and hence achieve a more scientific pricing framework.

Another interesting result arising from these models is in the use of the historical data in the credibility estimator. In the former model, $\bar{X}_i^{\text{Hom}}$ in (12) is simply the arithmetic average of the LCR's over the past T years, while in the latter model, $\bar{X}_i^{\text{Het}}$ in (18) is defined as the liability weighted average of historical LCR's. This gives rise to two different ways of averaging past LCR's, depending on the model assumptions.

This difference could have some important implications for ratemaking. In particular, as formally established in Appendix 1 (see *PA1*), under some additional technical assumptions, we have $\bar{X}_i^{\text{Het}} \geqslant \bar{X}_i^{\text{Hom}}$ almost surely; i.e. the liability weighted LCR is at least as large as the simple averaging LCR. More importantly, this also implies that ratemaking based on the liability weighted LCR should produce a risk premium that is higher than the corresponding rate based on the simple averaging LCR.

Additionally, since $\sigma_1^2(\Theta_i)$ and $\sigma_2^2(\Theta_i)$ can be interpreted as the conditional variance of the LCR (X_{ij}) and the indemnity (I_{ij}), respectively (recall (10) and (16)), it is reasonable to assume that the following relationship holds:

$$\sigma_1^2(\Theta_i) = \frac{1}{\bar{L}_i}\sigma_2^2(\Theta_i) \tag{22}$$

where $\bar{L}_i = \frac{1}{T}\sum_{j=1}^{T} L_{ij}$. Then we have:

$$\text{CreP}_i^{\text{Hom}} = \frac{a_i b_i}{a_i b_i + \frac{v^{\text{Hom}}}{Ta^{\text{Hom}}}}\bar{X}_i^{\text{Hom}} + \frac{a_i}{\frac{a_i b_i}{\frac{v^{\text{Hom}}}{Ta^{\text{Hom}}}} + 1}\mu_0 \tag{23}$$

$$\text{CreP}_i^{\text{Het}} = \frac{a_i b_i}{a_i b_i + \frac{v^{H}}{Ta^{\text{Hom}}}}\bar{X}_i^{\text{Het}} + \frac{a_i}{\frac{a_i b_i}{\frac{v^{\text{Hom}}}{Ta^{\text{Hom}}}} + 1}\mu_0 \tag{24}$$

where the $\bar{X}_i^{\text{Hom}}$ and $\bar{X}_i^{\text{Het}}$ are defined in (12) and (18), respectively. The above results suggest that if $\bar{X}_i^{\text{Het}} \geqslant \bar{X}_i^{\text{Hom}}$ then $\text{CreP}_i^{\text{Het}} \geqslant \text{CreP}_i^{\text{Hom}}$. Thus the credibility premium based on the liability weighted LCR's is more conservative than the corresponding premium based on the simple average LCR's.

6. Applications of credibility models

Recall that in Sections 3 and 4, an extensive analysis was conducted on a unique set of crop data, covering ten provinces in Canada over the years 1974-2009. Among all the plausible distributions that were investigated, the Erlang-mixture model was found to provide the best goodness of fit to the historical LCRs. Section 5 then described two modified versions of the Bühlmann-Straub credibility model, which may provide a better ratemaking framework for crop (re)insurance policies. Continuing to explore the

same set of data, this section provides additional empirical evidence regarding the appropriateness of the credibility models for pricing crop (re)insurance policies. In particular, Section 6.1 discusses how the credibility models can be used to integrate each province's historical data with the pooled historical data of all ten provinces to optimally determine the credibility premium (or equivalently the forecasted LCR) for year 2010. Then by using the province of MB as an example, an attempt is made in Section 6.2 to reproduce the empirically observed reinsurance experience for the year 2010.

6.1 Credibility premium with Erlang mixture distribution

The crop data for ten Canadian provinces (see Section 3) is used to conduct an empirical analysis for the credibility models described in the last section to predict the year 2010 LCR for each province. In our analysis, the prior distribution is assumed to be normal and the parameters are estimated using the maximum likelihood estimation method. For both homogeneous and heterogeneous risk models, the Erlang mixture model is calibrated to the LCR's. In addition, parameters such as a_i and b_i $(i = 1, 2, \ldots, d)$ are estimated based on the calibrated Erlang distribution, providing the prior information for each risk category (province). The forecasted credibility LCRs for year 2010 for both risk models (i.e. (11) and (17), respectively), together with the credibility factors (i.e. (13) and (19), respectively), are listed in Table VIII.

As a comparison, traditional Bülhmann-Straub model gives a credibility factor of 0.9870 for all risk categories (i.e. categories), while under the assumption of priori differences, the modified credibility models provide credibility factors (Z_i^{Hom} and Z_i^{Het}) that are different for each risk category.

The reported results clearly exemplifies the importance of the risk assumptions. The assumption of heterogeneity leads to higher predicted LCR's (relative to the homogeneous risk assumption) and thus is more conservative if the model is used for pricing (re)insurance contracts. Provinces such as Manitoba (MB), Ontario (ON), Saskatchewan (SK), New Brunswick (NB), and Quebec (QC) are the major crop producers in Canada, and it is of interest to note that switching from the homogeneous risk model to the heterogeneous risk model results in increased credibility factors for these provinces (thus giving more credits to their historical data). On the other hand, the credibility factors for provinces such as Nova Scotia (NS), Prince Edward Island

Province	Homogeneous model		Heterogeneous model	
	$Z_i^{\text{Hom}}{}_i$	PX_i^{Hom}	Z_i^{Het}	PX_i^{Het}
AB	0.9561	0.1333	0.9526	0.1405
MB	0.9574	0.0884	0.9618	0.1119
ON	0.9548	0.0553	0.9699	0.0639
BC	0.9777	0.0436	0.9774	0.0691
NB	0.9654	0.0930	0.9727	0.1121
NS	0.9729	0.0534	0.9597	0.0914
PEI	0.9737	0.0745	0.9517	0.1030
QC	0.9729	0.0748	0.9792	0.0903
SK	0.9553	0.1135	0.9743	0.1202
NFLD	0.9765	0.1156	0.9638	0.1538

Notes: Z_i^{Hom} and Z_i^{Het}, PX_i^{Hom} and PX_i^{Het} are credibility factors and predicted credibility adjusted LCRs for the ith province in the year 2010 under homogeneous and heterogeneous assumptions, respectively

Table VIII. Predicted credibility adjusted LCR's for the Year 2010

(PEI), and Newfoundland and Labrador (NFLD), which comprise a much smaller portion of the crop sector in Canada, result in decreased credibility factors. When switching from the homogeneous risk model to the heterogeneous risk model, the credibility factors decrease by 1.366, 2.285, and 1.309 percent, respectively. This means that the predicted LCR's for these provinces are giving more credit to information from the entire collective risk pool under a heterogeneous risk assumption.

6.2 Manitoba reinsurance example

The crop reinsurance treaty is written based on the LCR, and is typically a layer reinsurance contract structure (Porth *et al.*, 2013). For example, the reinsurance treaty for the province of Manitoba for the year 2010 has lower attachment LCR = 15 percent and upper level of LCR = 27.5 percent. This implies that if the observed LCR for the insured year is <15 percent, there is no reinsurance payout. If the observed LCR is >15 percent, then the reinsurers are liable for the losses in excess of 15 percent up to a maximum of 12.5 percent (which is the spread of the attachment points). The actual payout is then adjusted by the liability exposure forecasted at the inception of the contract. Manitoba Agriculture Service Corporation retains 10 percent of the liability so that the remaining 90 percent of the liability is ceded to private reinsurers. For year 2010, the liability exposure for the private reinsurers was $1,856,000,000 with reinsurance premium $28,700,000.

Using year 2010 reinsurance data from the province of Manitoba, the objective of this subsection is to consider various methods of pricing reinsurance contacts, and assess their effectiveness by relating the price to that observed in the market. To proceed, two critical assumptions are imposed. One is that the reinsurance premium P of the contract is assumed to be determined by the expectation premium principle, defined as:

$$P = (1 + \theta)E(L_R) \tag{25}$$

where θ is the loading factor and L_R is the reinsurer loss random variable. For year 2010 Manitoba reinsurance program, we have:

$$L_R = \max(X_{MB,2010} - 0.15, 0.125) \cdot L_{MB,2010} \tag{26}$$

where $X_{MB,2010}$ denotes Manitoba's LCR random variable in year 2010. Second we assume $\theta = 0.35$, which is consistent with the crop reinsurance market practice (Porth *et al.*, 2014).

Based on the above assumptions, the remaining task is to explore ways of determining the expectation in (25), in order to fully specify the reinsurance premium. Here we examine the following three methods:

- *Method I*: the random variable $X_{MB,\ 2010}$ in (26) is assumed to be modeled by the Erlang mixture model with its parameter values calibrated to the historical LCR data of Manitoba (see Section 4). $E(L_R)$ is then evaluated accordingly from the calibrated Erlang-mixture model.
- *Method II*: the credibility premium approach as described in Section 5.
- *Method III*: this approach is similar to the preceding credibility approach, but, with the important difference that instead of risk pooling from all ten provinces, the present approach exploits information only from a smaller subset of provinces.

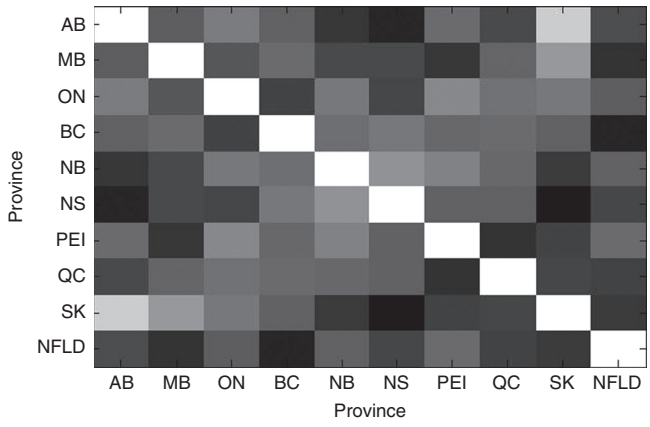
Here we use the K-means clustering technique to partition the ten provinces into groups with similar risk (as measured by the average risk). In our case, the ten provinces are classified into three groups according to their covariance coefficient matrix, as shown in Table IX and depicted in Figure 1. In Figure 1, the darker the squares signify risks that are less similar, and lighter squares show that the two corresponding risks are more similar. We note that in Table VIII Manitoba is grouped together with the provinces of Ontario, British Columbia, Nova Scotia, Prince Edward Island, and Quebec, and this group has the lowest risk exposure (average LCRs is 0.072). The credibility approach is then applied to this group for determining the reinsurance premium.

Note that Method I corresponds to the classical net premium approach, but, with risk loading. Both Methods II and III are based on the credibility approach, however, the latter method has the advantage that it relies on a smaller subset of data (i.e. Group I in Table IX). This reduces the amount of data needed to be collected and hence it is more manageable, especially where there is a large number of risk categories to begin with. Also, borrowing information from other risk categories with some similar traits appears to be more reasonable and credible.

The pricing results are summarized in Table X. For the credibility-based Methods II and III, both homogeneous and heterogeneous models are implemented. In addition to the reinsurance premium, the predicted credibility adjusted LCRs are also reported. The last column of the table gives the pricing error relative to the actual 2010 reinsurance premium, where a positive (negative) value indicates that it is larger (smaller) than the actual price. Using the 2010 price as the actual price, one immediate

Group	Provinces	Mean LCR
I	MB ON BC NS PEI QC	0.072
II	AB SK	0.120
III	NB NFLD	0.140

Table IX.
Classification for ten regions/provinces in Canada



Note: The darker squares signify risks that are less similar and lighter squares represent risks that are more alike

Figure 1.
Covariance coefficient matrix of ten regions/provinces in Canada

conclusion that can be drawn from the results is that net premium based only on the Erlang mixture model and the credibility premiums under the homogeneous assumptions are not satisfactory, as they all underestimate the observed premium by more than 10 per cent. These observations are quite surprising since the Erlang mixture model, provided the best goodness-of-fit, and a heavier tail. Therefore, it was expected that these methods should result in more conservative estimates of the reinsurance premiums. However, the reinsurance premiums calculated from these methods are still less than the observed market premium.

The impact of changing the assumption from homogeneous to heterogeneous is also highlighted. If we use all ten provinces in our heterogenous credibility model, then the resulting reinsurance premium is more conservative in that it is higher than the observed premium by almost 10 percent. On the other hand, if we adopt Method III which uses a smaller subset of provinces that are more alike, the underestimation error reduces from 13.9 to 4.0 percent by using the heterogenous credibility model, as opposed to the homogeneous model.

7. Conclusions

Crop insurance is faced with a major challenge of limited data which leads to concerns in pricing insurance and reinsurance contracts. The objective of this paper was to address some of the fundamental issues surrounding crop insurance ratemaking, from the perspective of the reinsurer, through the development of a scientific framework that can be consistently and widely adopted by the agricultural sector. We show by simulation that the current tools available for testing trends in time series cannot provide accurate information for small sample sizes as is in the case crop insurance. A unique data set comprised of the entire crop insurance sector in Canada was analyzed, and from this we show that the Erlang mixture distribution family, which is newly introduced in this paper for agricultural insurance ratemaking, captures the heavy tail property of the loss data better than single distribution models. In addition, the Erlang mixture model is shown to be robust under different trending methods, which is important in dealing with the difficult problem of deciding which trending process to use for small samples.

To enhance the crop (re)insurance pricing framework, a modified credibility model is introduced. Credibility premiums based on two different risk assumptions are calculated, including a homogeneous risk assumption and a heterogeneous risk assumption. Further, the "liability weighted" LCR is proposed to aggregate historical loss data. The results show that the heterogeneous credibility premium based on the liability weighted LCR is more conservative, and provides a more scientific and robust approach to enhance reinsurance pricing.

Table X.
Comparison of credibility factors, forecasted LCRs and reinsurance premiums under various pricing methods

Method		Z_{MB}	PX_{MB}	Reinsurance premium	Discrepancy (%)
I		–	–	$25,138,608	−12.4
II	Homogeneous	0.9548	0.0884	$24,906,567	−13.2
	Heterogeneous	0.9618	0.1119	$31,538,179	9.9
III	Homogeneous	0.9573	0.0877	$24,710,466	−13.9
	Heterogeneous	0.9614	0.0977	$27,548,036	−4.0

Note: The discrepancy column gives the pricing error relative to the year 2010 reinsurance premium of $28,700,000

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Appendix 1. Simple average LCR's vs liability weighted LCR's

For T years of observations, define two random variables P, and Q as:

$$P = \frac{1}{T}\sum_{i=1}^{T} X_i \tag{A1}$$

$$Q = \frac{\sum_{i=1}^{T} X_i L_i}{\sum_{i=1}^{T} L_i} \tag{A2}$$

where X_i, L_i $(i = 1, \ldots, T)$ are the LCR and liability in the ith year. Therefore, P and Q define two ways to aggregate historical losses, namely, P is the simple average LCR and Q is the liability weighted LCR. Then we have the following proposition:

PA1. In the probability space $(\Omega, \mathcal{F}, P)$, we define the following random variables:

(1) $X : \Omega \mapsto [0, 1]$ is LCR
(2) $Y : \Omega \mapsto R_+$ is crop yield
(3) $L : \Omega \mapsto R_+$ is liability
The relationship holds:

$$X = \frac{(L - Y)_+}{L} = \begin{cases} 0 & Y(\omega) \geqslant L(\omega), \omega \in \Omega \\ \frac{L-Y}{L} & Y(\omega) < L(\omega), \omega \in \Omega \end{cases}. \tag{A3}$$

We assume that liability is a function of yield, $f : R_+ \mapsto R_+$, namely, $L(\omega) = f(Y(\omega))$ for all $\omega \in \Omega_0$, where $\Omega_0 = \{\omega : Y(\omega) < L(\omega)\}$. Then if for all $x \in I = X(\Omega_0)$, function $f(x)$ satisfies the following:

(1) $f(x) \geqslant 0$ with $f(0) = 0$
(2) $f'(x) \geqslant 0$
(3) $f'(x) \geqslant f(x)/x$
Then $Q \geqslant P$ almost surely.

Proof. Let us start the proof by defining $L_{(i)}$ as the ith order statistic of random variables L, $i = 1, \ldots, T$, i.e.:

$$L_{(1)}(\omega) \leqslant L_{(2)}(\omega), \ldots, L_{(T)}(\omega), \quad \omega \in \Omega$$

If $\omega \in \Omega/\Omega_0$, then:

$$X_{(1)}(\omega) \leqslant X_{(2)}(\omega), \ldots, X_{(T)}(\omega), \quad \omega \in \Omega/\Omega_0$$

since $X_{(i)}(\omega) \in \Omega/\Omega_0 = 0$.

If $\omega \in \Omega_0$ let function $l : I \mapsto [0,1]$ be $l(y) = 1 - y/f(y)$. From the assumptions, we have $l'(y) \geqslant 0$, since:

$$l'(x) = \frac{yf'(y) - f(y)}{f^2(y)} \geqslant 0 \quad \forall y \in I$$

So $l(y)$ is an increasing function on I, which means that:

$$X_{(1)}(\omega) \leqslant X_{(2)}(\omega), \ldots, X_{(T)}(\omega), \quad \forall \omega \in \Omega_0$$

Therefore:

$$X_{(1)}(\omega) \leqslant X_{(2)}(\omega), \ldots, X_{(T)}(\omega), \quad \forall \omega \in \Omega$$

Namely:

$$\frac{X_{(1)}}{\sum_{i=1}^{T} L_{(i)}}(\omega) \leqslant \frac{X_{(2)}}{\sum_{i=1}^{T} L_{(i)}}(\omega), \ldots, \frac{X_{(T)}}{\sum_{i=1}^{T} L_{(i)}}(\omega), \quad \forall \omega \in \Omega$$

Then:

$$\sum_{i=1}^{T} \frac{1}{T} X_{(i)} \leqslant \sum_{i=1}^{T} L_{(i)} \frac{X_{(i)}}{\sum_{i=1}^{T} L_{(i)}} \quad a.s. \tag{A4}$$

which is equivalent to that $Q \geqslant P$ almost surely.

The inequality (A4) follows by directly applying the Chebyshev sum inequality; i.e. if:

$$a_1 \leqslant a_2, \ldots, \leqslant a_T \tag{A5}$$

$$b_1 \leqslant b_2, \ldots, \leqslant b_T \tag{A6}$$

Then:

$$\frac{1}{T} \sum_{i=1}^{T} a_i \sum_{i=1}^{T} b_i \leqslant \sum_{i}^{T} a_i b_i$$

Appendix 2. Deriving the credibility premiums

Credibility premiums search for the best linear combination of the past observations in the sense of minimizing the quadratic loss (for simplification of the notation, in the following derivation we suppress the subscript i, which indicates the risk category):

$$QL = E[(\mu(\Theta) - \alpha_0 - \sum_{j=1}^{T} \alpha_j X_j)^2]$$

Hence, the solution $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_N)'$ needs to satisfy the following for every $k = 1, 2, \ldots, T$:

$$\frac{\partial QL}{\partial \alpha_0} = (-2)\dot{E}\Big(\mu(\Theta) - \alpha_0 - \sum_{j=1}^{T} \alpha_j X_j\Big) = 0$$

$$\frac{\partial QL}{\partial \alpha_k} = (-2)\dot{E}\Big[X_k(\mu(\Theta) - \alpha_0 - \sum_{j=1}^{T} \alpha_j X_j)\Big] = 0$$

Namely:

$$E(\mu(\Theta)) = \alpha_0 + \sum_{j=1}^{T} \alpha_j E(X_j)$$

$$E(\mu(\Theta)X_k) = \alpha_0 + \sum_{j=1}^{T} \alpha_j E(X_j X_k)$$

Also note that:

$$\begin{aligned} E(\mu(\Theta)) &= E(E(X_{T+1}|\Theta)) = E(X_{T+1}) \\ E(\mu(\Theta)X_k) &= E(E(\mu(\Theta)X_k|\Theta)) \\ &= E(\mu(\Theta)E(X_k|\Theta)) \\ &= E(E(X_{T+1}|\mu(\Theta))E(X_k|\Theta)) \\ &= E(X_{T+1}X_k) \end{aligned}$$

Thus:

$$E(X_{T+1}) = \alpha_0 + \sum_{j=1}^{T} \alpha_j E(X_j) \tag{A7}$$

$$Cov(X_{T+1}, X_k) = \sum_{j=1}^{T} \alpha_j Cov(X_j, X_k) \tag{A8}$$

The above two equations are called "Normal Equations."

For M1:

$$\begin{aligned} Cov(X_{T+1}, X_k) &= Cov(E(X_{T+1}|\Theta), E(X_k|\Theta)) + E(Cov(X_{T+1}|\Theta, X_k|\Theta) \\ &= a^2 a^{\text{Hom}} \end{aligned}$$

$$\begin{aligned} Cov(X_j, X_k) &= Cov(E(X_j|\Theta), E(X_k|\Theta)) + E(Cov(X_j|\Theta, X_k|\Theta) \\ &= a^2 a^{\text{Hom}} + \frac{a}{b} v^{\text{Hom}} \delta_{kj} \end{aligned}$$

where:

$$\delta_{kj} = \begin{cases} 0, & k = j \\ 1, & \text{else} \end{cases}$$

From (A7):

$$a\mu_0 = \alpha_0 + a\mu_0 \sum_{j=1}^{T} \alpha_j \tag{A9}$$

From (A8):

$$\begin{aligned} a^2 a^{\text{Hom}} &= \sum_{j=1, j\neq k}^{T} \alpha_j a^2 a^{\text{Hom}} + a_k (a^2 a^{\text{Hom}} + \frac{a}{b} v^{\text{Hom}}) \\ &= \sum_{j=1}^{T} a^2 a^{\text{Hom}} \alpha_j + \frac{a}{b} v^{\text{Hom}} \alpha_k \end{aligned}$$

Hence:

$$\alpha_0 = \frac{\frac{v^{\text{Hom}}}{a^{\text{Hom}}}}{Tab + \frac{v^{\text{Hom}}}{a^{\text{Hom}}}} a\mu_0, \quad \alpha_j = \frac{ab}{Tab + \frac{v^{\text{Hom}}}{a^{\text{Hom}}}}$$

Thus, we finally get:

$$\text{CreP}^{\text{Hom}} = Z^{\text{Hom}} \bar{X}^{\text{Hom}} + (1 - Z^{\text{Hom}})\mu$$

where:

$$Z^{\text{Hom}} = \frac{Tab}{Tab + \frac{v^{\text{Hom}}}{a^{\text{Hom}}}}, \quad \mu = a\mu_0 \quad \text{and} \quad \bar{X}^{\text{Hom}} = \frac{1}{T}\sum_{j=1}^{T} X_j$$

For M2, similarly:

$$\begin{aligned} Cov(X_{T+1}, X_k) &= Cov(E(X_{T+1}|\Theta), E(X_k|\Theta)) + E(Cov(X_{T+1}|\Theta, X_k|\Theta) \\ &= Var(E(X_k|\Theta)) + 0 \\ &= a^2 a^{\text{Het}} \end{aligned}$$

$$\begin{aligned} Cov(X_j, X_k) &= Cov(E(X_j|\Theta), E(X_k|\Theta)) + E(Cov(X_j|\Theta, X_k|\Theta) \\ &= a^2 a^{\text{Het}} + \frac{a}{bL_k} v^{\text{Het}} \delta_{kj} \end{aligned}$$

From (A7):

$$a\mu_0 = \alpha_0 + a\mu_0 \sum_{j=1}^{T} \alpha_j \tag{A10}$$

From (A8):

$$\begin{aligned} a^2 a^{\text{Het}} &= \sum_{j=1, j\neq k}^{T} \alpha_j a^2 a^{\text{Het}} + a_k (a^2 a^{\text{Het}} + \frac{a}{bL_j} v^{\text{Het}}) \\ &= \sum_{j=1}^{T} a^2 a^{\text{Het}} \alpha_j + \frac{a}{bL_j} v^{\text{Het}} \alpha_k \end{aligned}$$

Hence, by defining $L = \sum_{j=1}^{T} L_j$, we have:

$$\alpha_0 = \frac{\frac{v^{\text{Het}}}{a^{\text{Het}}}}{Lab + \frac{v^{\text{Het}}}{a^{\text{Het}}}} a\mu_0, \quad \alpha_k = \frac{L_k ab}{Lab + \frac{v^{\text{Het}}}{a^{\text{Het}}}}$$

Thus, we finally get:

$$\text{CreP}^{\text{Het}} = Z^{\text{Het}} \bar{X}^{\text{Het}} + (1 - Z^{\text{Het}})\mu$$

where:

$$Z^{\text{Het}} = \frac{Lab}{Lab + \frac{v^{\text{Het}}}{a^{\text{Het}}}}$$

$\mu = \alpha\mu_0$ and

$$\bar{X}^{\text{Het}} = \sum_{j=1}^{T} \frac{L_j}{L} X_j$$

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